# Revisiting Margins

Enrique Pinzón

StataCorp LP

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Barcelona

# Introduction

- I am going to use today as an excuse to talk about `margins` and how I visualize models and their interpretation
- As many Stata tools, `margins` is powerful yet underutilized.
- `margins` helps us interpret and use the results of our models.

- Review concepts to stablish a common ground
  - Linear cross-sectional models
  - Nonlinear cross-sectional models
- Present preliminary results of an article I have been working on
  - Linear panel-data models
  - Nonlinear panel-data models

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# Object of Interest

- Interpreting the results from a model depend on the object of interest
- It is rarely the case that the estimated parameters are the object of interest
- We will illustrate this with linear regression

# Linear Regression Model

- The model is given by

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_{i1} + \ldots \beta_k x_{ik} + \varepsilon_i \\
E\left(\varepsilon_i | x_{i1} \ldots x_{ik}\right) &= 0 \\
E\left(y_i | x_{i1} \ldots x_{ik}\right) &= \beta_0 + \beta_1 x_{i1} + \ldots \beta_k x_{ik}
\end{aligned}
$$

- We would like to now how a change in one of the regressors affects $E\left(y_i | x_{i1} \ldots x_{ik}\right)$. For example, for a continuous $x_k$ we have that :

$$
\frac{\partial E\left(y_i | x_{i1} \ldots x_{ik}\right)}{\partial x_k} = \beta_k
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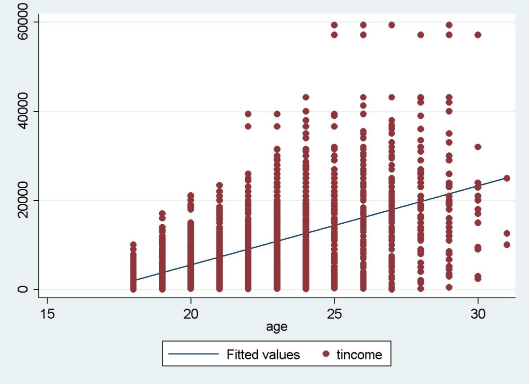
$$\frac{\partial E\left(y_i | x_{i1} \ldots x_{ik}\right)}{\partial x_k} = \beta_k$$

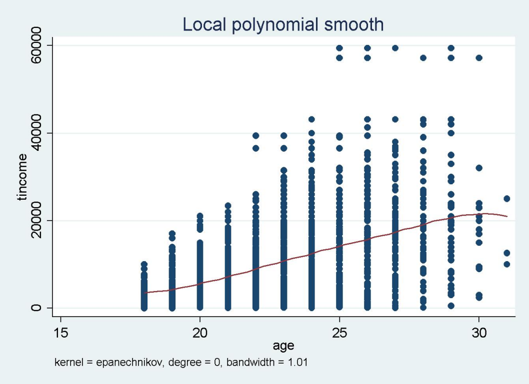
# Graphical Representation

$$
\begin{aligned}
Income &= \beta_0 + \beta_1 Age + \beta_2 Hispanic + \beta_3 Male + \varepsilon \\
E\left(Income | Age, Hispanic = Male = 1\right) &= \delta + \beta_1 Age \\
\delta &\equiv \beta_0 + \beta_2 + \beta_3
\end{aligned}
$$



# Graphical Representation

$$
\begin{aligned}
\textit{Income} &= g\left(\textit{Age}, \textit{Hispanic}, \textit{Male}\right) + \varepsilon \\
& E\left(\textit{Income} \mid \textit{Age}, \textit{Hispanic} = \textit{Male} = 1\right)
\end{aligned}
$$



# Interaction terms

- If our model is of the form

$$Income = \beta_0 + \beta_1 Age + \beta_2 Hispanic + \beta_3 Male + \beta_4 Age^2 + \varepsilon$$

- The marginal effects are of the form

$$\frac{\partial E\left(Income|Age, Hispanic, Male\right)}{\partial Age} = \beta_1 + 2\beta_4 Age$$

- `margins` helps us obtain the correct value and standard error for the average marginal effect

# Interaction terms

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- `margins` helps us obtain the correct value and standard error for the average marginal effect

```
. regress tincome i.hispanic i.male c.age##c.age, noheader
```

| tincome | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| 1.hispanic | -13.25757 | 118.8548 | -0.11 | 0.911 | -246.218 | 219.7029 |
| 1.male | 2425.08 | 85.18172 | 28.47 | 0.000 | 2258.12 | 2592.04 |
| age | 908.2874 | 198.5842 | 4.57 | 0.000 | 519.054 | 1297.521 |
| c.age#c.age | 17.16441 | 4.26985 | 4.02 | 0.000 | 8.795324 | 25.5335 |
| _cons | -21689.52 | 2283.89 | -9.50 | 0.000 | -26166.04 | -17213 |

```
. quietly generate margin = _b[age] + 2*_b[c.age#c.age]*age
. summarize margin
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| margin | 30213 | 1688.333 | 100.9031 | 1526.206 | 1972.481 |

```
. margins, dydx(age)
Average marginal effects                        Number of obs   =      30213
Model VCE    : OLS
Expression   : Linear prediction, predict()
dy/dx w.r.t. : age
```

| | dy/dx | Delta-method Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| age | 1688.333 | 15.03678 | 112.28 | 0.000 | 1658.861 | 1717.806 |

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# Probit and Logit Models

- Probit and Logit models are given by:

$$y_i = \begin{cases} 1 & \text{if} \quad y_i^* = \beta_0 + \beta_1 x_{i1} + \ldots \beta_k x_{ik} + \varepsilon_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

- The assumptions we make on the distribution of $\varepsilon_i$ give us different models.
- The object of interest is $P\left(y_i | x_{i1} \ldots x_{ik}\right)$

# Probit

$$\frac{\partial P\left(y_i | x_{i1} \ldots x_{ik}\right)}{\partial x_j} = \phi\left(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_k x_{ik}\right)\beta_j$$

- Average marginal effect

$$\frac{1}{N}\sum_{i=1}^{N}\phi\left(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_k x_{ik}\right)\beta_j$$

- Marginal effect at a point, for example $\bar{x}$

$$\phi\left(\beta_0 + \beta_1 \bar{x}_1 + \ldots + \beta_k \bar{x}_k\right)\beta_j$$

# Probit

$$\frac{\partial P\left(y_i|x_{i1}\ldots x_{ik}\right)}{\partial x_j} = \phi\left(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_k x_{ik}\right)\beta_j$$

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$$\phi\left(\beta_0 + \beta_1 \bar{x}_1 + \ldots + \beta_k \bar{x}_k\right)\beta_j$$

# Probability of Never Being Married

```
. probit nevermarried age i.male i.college, nolog
Probit regression                               Number of obs   =      30213
                                                LR chi2(3)      =    6018.41
                                                Prob > chi2     =     0.0000
Log likelihood = -15229.266                     Pseudo R2       =     0.1650
```

| nevermarried | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| age | -.2302782 | .0033003 | -69.77 | 0.000 | -.2367467 | -.2238097 |
| 1.male | .3077872 | .0165872 | 18.56 | 0.000 | .2752768 | .3402976 |
| 1.college | .354033 | .0208055 | 17.02 | 0.000 | .313255 | .3948109 |
| _cons | 5.655506 | .0757096 | 74.70 | 0.000 | 5.507118 | 5.803894 |

```
. estimates store probit
```

# Average Marginal Effects

```
. margins, dydx(age)
Average marginal effects                          Number of obs   =      30213
Model VCE     : OIM
Expression    : Pr(nevermarried), predict()
dy/dx w.r.t. : age

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |  -.0650363   .0006976   -93.23   0.000    -.0664036    -.063669
------------------------------------------------------------------------------
```

```
. margins, dydx(age) atmeans
Conditional marginal effects                      Number of obs   =      30213
Model VCE     : OIM
Expression    : Pr(nevermarried), predict()
dy/dx w.r.t. : age
at            : age             =    22.72277 (mean)
                0.male          =    .4962764 (mean)
                1.male          =    .5037236 (mean)
                0.college       =    .7830404 (mean)
                1.college       =    .2169596 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |  -.0741411   .0010415   -71.19   0.000    -.0761823   -.0720998
------------------------------------------------------------------------------
```

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| | dy/dx | Delta-method Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| age | -.0650363 | .0006976 | -93.23 | 0.000 | -.0664036 | -.063669 |

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|---|---|---|---|---|---|---|
| age | -.0741411 | .0010415 | -71.19 | 0.000 | -.0761823 | -.0720998 |

# Logit vs. Probit

- Average marginal effects, marginal effects at a point, and predicted probabilities are similar for logit and probit models.
- The reason is that the logistic and normal distributions are very similar.
- If the objects of interest are the ones mentioned above, the use of one or the other is a fruit of custom.

# Logit vs. Probit: Point Estimates

```
. quietly logit nevermarried age i.male i.college, nolog
. estimates store logit
. estimates table logit probit
```

| Variable | logit | probit |
|---|---|---|
| age | -.38456626 | -.23027823 |
| male | | |
| 1 | .52122078 | .30778721 |
| college | | |
| 1 | .5892684 | .35403299 |
| _cons | 9.4333826 | 5.6555062 |

# Logit vs. Probit: Average Marginal Effects

```
. quietly margins, dydx(age) post

. estimates store dydxlog

. estimates restore probit
(results probit are active now)

. quietly margins, dydx(age) post

. estimates store dydxprob

. estimates table dydx*
```

| Variable | dydxlog | dydxprob |
|---|---|---|
| age | -.06406436 | -.06503633 |

# Logit vs. Probit: Marginal Effects at Means

```
. estimates restore logit
(results logit are active now)

. quietly margins, dydx(age) atmeans post

. estimates store atlog

. estimates restore probit
(results probit are active now)

. quietly margins, dydx(age) atmeans post

. estimates store atprob

. estimates table at*
```

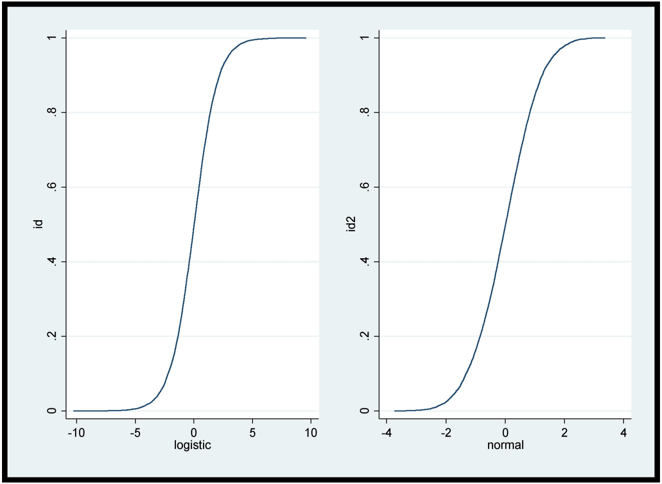
| Variable | atlog | atprob |
|---|---|---|
| age | -.07258275 | -.07414107 |

# Logit vs. Probit: Predicted Probabilities

```
 . estimates restore logit
(results logit are active now)

. predict prlogit
(option pr assumed; Pr(nevermarried))

. estimates restore probit
(results probit are active now)

. predict prprobit
(option pr assumed; Pr(nevermarried))

. summarize pr*
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| prlogit | 30213 | .7083044 | .2000549 | .0766916 | .9622073 |
| prprobit | 30213 | .7102447 | .2006532 | .0690214 | .9739339 |

# Logit vs. Probit: Graphically



# Panel Data

- Allows us to model and understand individual heterogeneity
- Follow effects of policy or interventions across time
- Most of our intuition from cross-sectional data (objects of interest) translate to panel-data

# Object of Interest: Linear Panel-Data Model

- The linear panel-data model is given by:

$$
\begin{aligned}
y_{it} &= \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + \alpha_i + \varepsilon_{it} \\
E(\varepsilon_{it}|X, \alpha_i) &= 0
\end{aligned}
$$

- Random Effects: $E(\alpha_i|x_{1it} \ldots x_{kit}) = 0$

$$E(y_{it}|x_{1it} \ldots x_{kit}) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit}$$

- Fixed Effects: $E(\alpha_i|x_{1it} \ldots x_{kit}) \neq 0$

$$E(y_{it}|x_{1it} \ldots x_{kit}) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + E(\alpha_i|x_{1it} \ldots x_{kit})$$

# Object of Interest: Linear Panel-Data Model

- The linear panel-data model is given by:

$$\begin{aligned} y_{it} &= \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + \alpha_i + \varepsilon_{it} \\ E\left(\varepsilon_{it}|X, \alpha_i\right) &= 0 \end{aligned}$$

- Random Effects: $E\left(\alpha_i|x_{1it} \ldots x_{kit}\right) = 0$

$$E\left(y_{it}|x_{1it} \ldots x_{kit}\right) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit}$$

- Fixed Effects: $E\left(\alpha_i|x_{1it} \ldots x_{kit}\right) \neq 0$

$$E\left(y_{it}|x_{1it} \ldots x_{kit}\right) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + E\left(\alpha_i|x_{1it} \ldots x_{kit}\right)$$

# Object of Interest: Linear Panel-Data Model

- The linear panel-data model is given by:

$$\begin{aligned} y_{it} &= \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + \alpha_i + \varepsilon_{it} \\ E\left(\varepsilon_{it}|X, \alpha_i\right) &= 0 \end{aligned}$$

- Random Effects: $E\left(\alpha_i|x_{1it} \ldots x_{kit}\right) = 0$

$$E\left(y_{it}|x_{1it} \ldots x_{kit}\right) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit}$$

- Fixed Effects: $E\left(\alpha_i|x_{1it} \ldots x_{kit}\right) \neq 0$

$$E\left(y_{it}|x_{1it} \ldots x_{kit}\right) = \beta_0 + \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + E\left(\alpha_i|x_{1it} \ldots x_{kit}\right)$$

# Fixed-Effects Solution

- Transform variables:

$$
\begin{aligned}
\bar{y}_i &= \frac{1}{T}\sum_{t=1}^{T} y_{it} \\
&= \beta_0 + \beta_1 \bar{x}_{1i} + \ldots + \beta_k \bar{x}_{ki} + \alpha_i + \bar{\varepsilon}_i
\end{aligned}
$$

- Generate new varible

$$
\begin{aligned}
y_{it} - \bar{y}_i &= (\beta_0 - \beta_0) + \beta_1 (x_{1it} - \bar{x}_{1i}) + \ldots + \beta_k (x_{kit} - \bar{x}_{ki}) \\
&+ (\alpha_i - \alpha_i) + (\varepsilon_{it} - \bar{\varepsilon}_i) \\
\ddot{y}_{it} &= \beta_1 \ddot{x}_{1it} + \ldots + \beta_k \ddot{x}_{kit} + \ddot{\varepsilon}_{it} \\
\ddot{y}_{it} &\equiv y_{it} - \bar{y}_i
\end{aligned}
$$

# Fixed-Effects Solution

- Transform variables:

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$$
$$= \beta_0 + \beta_1 \bar{x}_{1i} + \ldots + \beta_k \bar{x}_{ki} + \alpha_i + \bar{\varepsilon}_i$$

- Generate new varible

$$y_{it} - \bar{y}_i = (\beta_0 - \beta_0) + \beta_1 (x_{1it} - \bar{x}_{1i}) + \ldots + \beta_k (x_{kit} - \bar{x}_{ki})$$
$$+ \ (\alpha_i - \alpha_i) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$
$$\ddot{y}_{it} = \beta_1 \ddot{x}_{1it} + \ldots + \beta_k \ddot{x}_{kit} + \ddot{\varepsilon}_{it}$$
$$\ddot{y}_{it} \equiv y_{it} - \bar{y}_i$$

# Object of Interest: Conditional Expectation

- The above procedure gives us a consistent estimator of the parameters $\hat{\beta}_1, \ldots, \hat{\beta}_k$
- But:

$$\begin{aligned} \hat{E}\left(y_{it}|x_{1it} \ldots x_{kit}\right) &= \hat{\beta}_1 x_{1it} + \ldots + \hat{\beta}_k x_{kit} \\ &\nrightarrow E\left(y_{it}|x_{1it} \ldots x_{kit}\right) \end{aligned}$$

- The reason is that we do not have an estimate of:

$$E\left(\alpha_i|x_{1it} \ldots x_{kit}\right)$$

# Object of Interest: Marginal Effects

- Likewise

$$\frac{\partial \widehat{E\left(y_{it}|x_{1it}\ldots x_{kit}\right)}}{\partial x_{kit}} = \hat{\beta}_k$$

$$\nrightarrow \frac{\partial E\left(y_{it}|x_{1it}\ldots x_{kit}\right)}{\partial x_{kit}}$$

- The reason is that we do not have an estimate for

$$\frac{\partial E\left(\alpha_i|x_{1it}\ldots x_{kit}\right)}{\partial x_{kit}}$$

# Proposed Solution: Mundlak-Chamberlain Approach

- Propose a model for

$$E\left(\alpha_i | x_{1it} \ldots x_{kit}\right)$$

- We do the same in regression were we model $E\left(y_i | x_{i1} \ldots x_{ik}\right)$
- The model is of the form

$$\begin{aligned}
\alpha_i &= \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik} + \nu_i \\
E\left(\nu_i | x_{i1} \ldots x_{ik}\right) &= 0 \\
E\left(\alpha_i | x_{1it} \ldots x_{kit}\right) &= \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik}
\end{aligned}$$

- This implies that we are back into the random-effects framework and:

$$E\left(y_{it} | x_{1it} \ldots x_{kit}\right) = \beta_1 x_{1it} + \ldots + \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik}$$

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# Fixed-Effects Estimation

```
. xtreg tincome tenure c.age##c.age i.hispanic i.male, fe
note: 1.hispanic omitted because of collinearity
note: 1.male omitted because of collinearity
Fixed-effects (within) regression               Number of obs      =     30213
Group variable: id                              Number of groups   =      4343
R-sq:  within  = 0.4635                         Obs per group: min =         4
       between = 0.2218                                        avg =       7.0
       overall = 0.3644                                        max =         9
                                                F(3,25867)         =   7449.50
corr(u_i, Xb)  = -0.1075                        Prob > F           =    0.0000

------------------------------------------------------------------------------
     tincome |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      tenure |   20.00194   .5618164    35.60   0.000     18.90075    21.10313
         age |   70.32832   159.8937     0.44   0.660    -243.0722    383.7288
             |
 c.age#c.age |   36.37289   3.476712    10.46   0.000     29.55834    43.18744
             |
  1.hispanic |          0  (omitted)
      1.male |          0  (omitted)
       _cons |  -13092.78   1826.164    -7.17   0.000    -16672.17     -9513.4
-------------+----------------------------------------------------------------
     sigma_u |  4941.7061
     sigma_e |  5614.1177
         rho |   .4365569   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(4342, 25867) =     5.24         Prob > F = 0.0000
```

# Mundlak-Chamberlain Estimates

```
. quietly xtreg tincome tenure c.age##c.age, fe
. estimates store fe
. local muestra = e(sample)
. by id: egen mtenure           = mean(tenure)   if `muestra´
. by id: egen double mage       = mean(age)      if `muestra´
. by id: egen mmale             = mean(male)     if `muestra´
. by id: egen mhispanic         = mean(hispanic) if `muestra´
. by id: egen double mage2      = mean(age^2)    if `muestra´
. quietly xtreg tincome tenure c.age##c.age mtenure mhispanic mmale mage mage2
. estimates store mundlak
```

# Mundlak-Chamberlain vs. Fixed Effects

```
. estimates table fe mundlak
```

| Variable | fe | mundlak |
|---|---|---|
| tenure | 20.001943 | 20.001943 |
| age | 70.328319 | 70.328319 |
| c.age#c.age | 36.372886 | 36.372886 |
| mtenure | | 16.562624 |
| mhispanic | | -259.96981 |
| mmale | | 2248.8968 |
| mage | | 3504.7492 |
| mage2 | | -91.017878 |
| _cons | -13092.784 | -47461.511 |

# Testing

```
. estimates restore mundlak
(results mundlak are active now)

. test mtenure mage mmale mhispanic mage2

 ( 1)  mtenure = 0
 ( 2)  mage = 0
 ( 3)  mmale = 0
 ( 4)  mhispanic = 0
 ( 5)  mage2 = 0

           chi2(  5) =  700.54
         Prob > chi2 =    0.0000
```

# Objects of Interest

- Conditional expectation

```
. predict xbeta
(option xb assumed; fitted values)
```

- For the marginal effects

$$\frac{\partial E\left(y_{it}|x_{1it}\ldots x_{kit}\right)}{\partial x_{kit}} = \beta_k + \frac{1}{T}\theta_k$$

- For our example

$$\beta_{age} + (1/10)\beta_{mage} + 2age\beta_{agesq} + (2/10)age\beta_{agesq}$$

# Average Marginal Effects

```
. margins, ///
> expression(_b[age] +  (1/10)*_b[mage] + _b[c.age#c.age]*2*age + ///
> (1/10)*_b[mage2]*2*age) over(year)
Warning: expression() does not contain predict() or xb().
Predictive margins                          Number of obs   =      30213
Model VCE    : Conventional
Expression   : _b[age] + (1/10)*_b[mage] + _b[c.age#c.age]*2*age + (1/10)*_b[mage2]*2*age
over         : year
```

| | Margin | Delta-method Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| year | | | | | | |
| 79 | 1482.805 | 30.85434 | 48.06 | 0.000 | 1422.332 | 1543.279 |
| 80 | 1500.187 | 28.76721 | 52.15 | 0.000 | 1443.804 | 1556.569 |
| 81 | 1524.764 | 25.90806 | 58.85 | 0.000 | 1473.985 | 1575.542 |
| 82 | 1548.672 | 23.26417 | 66.57 | 0.000 | 1503.075 | 1594.269 |
| 83 | 1583.816 | 19.73891 | 80.24 | 0.000 | 1545.128 | 1622.503 |
| 84 | 1637.464 | 15.77607 | 103.79 | 0.000 | 1606.543 | 1668.384 |
| 85 | 1686.49 | 14.69991 | 114.73 | 0.000 | 1657.678 | 1715.301 |
| 86 | 1740.62 | 16.91648 | 102.89 | 0.000 | 1707.464 | 1773.775 |
| 87 | 1794.102 | 21.56305 | 83.20 | 0.000 | 1751.839 | 1836.365 |
| 88 | 1848.186 | 27.50478 | 67.20 | 0.000 | 1794.277 | 1902.094 |

# Simulations for Linear Panel Data Models

Table: Averages Marginal Effects

Simulations for `xtreg` with $N = 400$ and $T = 5$

| ESTIMATOR | TRUE EFFECTS | ESTIMATED EFFECTS | COVERAGE |
|---|---|---|---|
| MUNDLAK | $x_1$ = -.6 | -.602 | .952 |
| | $x_2$ = .6 | .599 | .946 |
| | $x_3^*$ = .36 | .363 | .939 |
| | $x_4$ = .36 | .36 | .951 |
| R.EFFECTS | $x_1$ = -.6 | -.602 | .949 |
| | $x_2$ = .6 | .599 | .947 |
| | $x_3^*$ = .36 | .314 | .841 |
| | $x_4$ = .36 | .311 | .499 |
| F.EFFECTS | $x_1$ = -.6 | . | . |
| | $x_2$ = .6 | . | . |
| | $x_3^*$ = .36 | .301 | .784 |
| | $x_4$ = .36 | .299 | .32 |

*The variable $x_3$ is discrete. Thus, this is a population treatment effect and the estimates are ATEs.

# Nonlinear Panel-Data Models: Probit and Logit

- If we are interested in average treatment effects and average effects over the population, population averaged models are best
- Probit panel-data models for fixed-effects are not available in Stata
- Logit fixed-effects models will drop panels that have invariant outcomes
- The main problem is that we cannot get rid of the time-invariant random disturbance
- No numerical integration

# Objects of Interest

- The models are given by

$$y_{it} = \begin{cases} 1 & \text{if} \quad y_{it}^* = \beta_0 + \beta_1 x_{1it} + \ldots \beta_k x_{kit} + \alpha_i + \varepsilon_{it} > 0 \\ 0 & \text{otherwise} \end{cases}$$

- The object of interest for the probit fixed-effects model is

$$P\left(y_{it}|x_{1it}, \ldots, x_{ikt}, \alpha_i\right) = \Phi\left(\beta_0 + \beta_1 x_{1it} + \ldots \beta_k x_{kit} + \alpha_i\right)$$

- Using the Mundlak-Chamberlain approach

$$\Phi\left(\beta_0 + \beta_1 x_{1it} + \ldots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik}\right)$$

- The marginal effects for $x_{1it}$:

$$\phi\left(\beta_0 + \beta_1 x_{1it} + \ldots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik}\right)\left(\beta_1 + \frac{1}{T}\theta_1\right)$$

- For a discrete $x_{1it}$

$$\begin{aligned} & \Phi\left(\beta_0 + \beta_1 + \ldots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \ldots + \theta_k \bar{x}_{ik}\right) \\ - \quad & \Phi\left(\beta_0 + \ldots + \beta_k x_{kit} + \ldots + \theta_k \bar{x}_{ik}\right) \end{aligned}$$

# Objects of Interest

- The models are given by

$$y_{it} = \begin{cases} 1 & \text{if} \quad y_{it}^* = \beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \alpha_i + \varepsilon_{it} > 0 \\ 0 & \text{otherwise} \end{cases}$$

- The object of interest for the probit fixed-effects model is

$$P\left(y_{it} | x_{1it}, \dots, x_{ikt}, \alpha_i\right) = \Phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \alpha_i\right)$$

- Using the Mundlak-Chamberlain approach

$$\Phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right)$$

- The marginal effects for $x_{1it}$:

$$\phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right)\left(\beta_1 + \frac{1}{T}\theta_1\right)$$

- For a discrete $x_{1it}$

$$\begin{aligned} & \Phi\left(\beta_0 + \beta_1 + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right) \\ - \quad & \Phi\left(\beta_0 + \dots + \beta_k x_{kit} + \dots + \theta_k \bar{x}_{ik}\right) \end{aligned}$$

# Objects of Interest

- The models are given by

$$y_{it} = \begin{cases} 1 & \text{if} \quad y^*_{it} = \beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \alpha_i + \varepsilon_{it} > 0 \\ 0 & \text{otherwise} \end{cases}$$

- The object of interest for the probit fixed-effects model is

$$P\left(y_{it} | x_{1it}, \dots, x_{ikt}, \alpha_i\right) = \Phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \alpha_i\right)$$

- Using the Mundlak-Chamberlain approach

$$\Phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right)$$

- The marginal effects for $x_{1it}$:

$$\phi\left(\beta_0 + \beta_1 x_{1it} + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right)\left(\beta_1 + \frac{1}{T}\theta_1\right)$$

- For a discrete $x_{1it}$

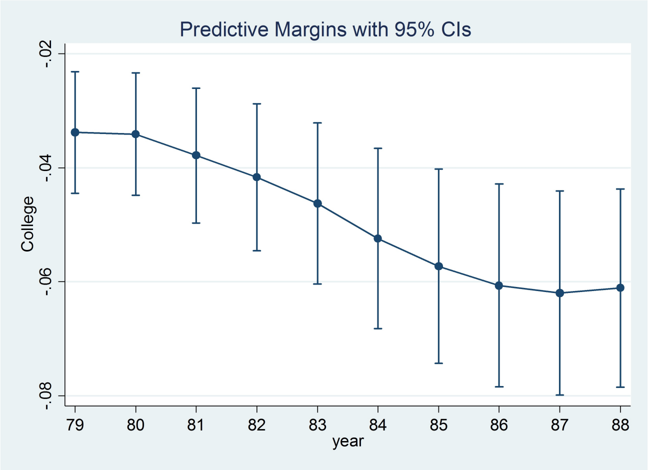
$$\begin{aligned} & \Phi\left(\beta_0 + \beta_1 + \dots \beta_k x_{kit} + \theta_1 \bar{x}_{i1} + \dots + \theta_k \bar{x}_{ik}\right) \\ - \quad & \Phi\left(\beta_0 + \dots + \beta_k x_{kit} + \dots + \theta_k \bar{x}_{ik}\right) \end{aligned}$$

# Probit Example

```
 . quietly xtprobit nevermarried age i.college mage mmale mcollege, nolog pa
. generate xb =  _b[age]*age +  _b[mage]*mage + _b[mmale]*mmale + _b[_cons] ///
> + _b[mcollege]*mcollege
. margins, over(year) ///
>                 expression(normal(xb +  _b[1.college])- ///
>                 normal(xb))
Warning: expression() does not contain predict() or xb().
Predictive margins                                Number of obs   =      30213
Model VCE     : Conventional
Expression    : normal(xb + _b[1.college])- normal(xb)
over          : year
```

| | Margin | Delta-method Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| year | | | | | | |
| 79 | -.0337846 | .0054447 | -6.21 | 0.000 | -.0444561 | -.0231131 |
| 80 | -.0341023 | .0054792 | -6.22 | 0.000 | -.0448412 | -.0233633 |
| 81 | -.0378459 | .0060266 | -6.28 | 0.000 | -.0496579 | -.0260339 |
| 82 | -.0416477 | .00657 | -6.34 | 0.000 | -.0545246 | -.0287707 |
| 83 | -.0462502 | .0072155 | -6.41 | 0.000 | -.0603923 | -.032108 |
| 84 | -.0524056 | .0080612 | -6.50 | 0.000 | -.0682052 | -.036606 |
| 85 | -.0572666 | .0086965 | -6.59 | 0.000 | -.0743114 | -.0402218 |
| 86 | -.0606502 | .0090759 | -6.68 | 0.000 | -.0784386 | -.0428617 |
| 87 | -.0619659 | .0091375 | -6.78 | 0.000 | -.079875 | -.0440568 |
| 88 | -.0610928 | .0088768 | -6.88 | 0.000 | -.078491 | -.0436947 |

# Logit vs. Probit: Graphically



# Simulations for Linear Panel Data Models

Table: Averages Effects Over Individuals for year 2009:

Simulation Results for `xtprobit` with $N = 400$ and $T = 5$

| ESTIMATOR | TRUE EFFECTS | ESTIMATED EFFECTS | COVERAGE |
|---|---|---|---|
| PA MUNDLAK | $x_1$ = -.132 | -.132 | .951 |
| | $x_2$ = .132 | .132 | .937 |
| | $x_3^*$ = .067 | .069 | .939 |
| | $x_4$ = .079 | .079 | .947 |
| R.EFFECTS | $x_1$ = -.132 | -.159 | .572 |
| | $x_2$ = .132 | .159 | .568 |
| | $x_3^*$ = .067 | .086 | .859 |
| | $x_4$ = .079 | .086 | .896 |
| PA | $x_1$ = -.132 | -.133 | .949 |
| | $x_2$ = .132 | .132 | .933 |
| | $x_3^*$ = .067 | .072 | .953 |
| | $x_4$ = .079 | .071 | .827 |

*The variable $x_3$ is discrete. Thus, this is a population treatment effect and the estimates are ATEs.

# Conclusions

- I presented a new way to obtain marginal effects using a Mundlak-Chamberlain approach
- Along the way we talked about margins and about objects of interest being the guiding principle to use and interpret estimation results
- Thank you

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