Introduction to Markov-switching regression models using the `mswitch` command

Gustavo Sánchez

StataCorp

May 18, 2016
Aguascalientes, México

Introduction to Markov-switching regression models using the `mswitch` command

Gustavo Sánchez

StataCorp

May 18, 2016
Aguascalientes, México

Outline

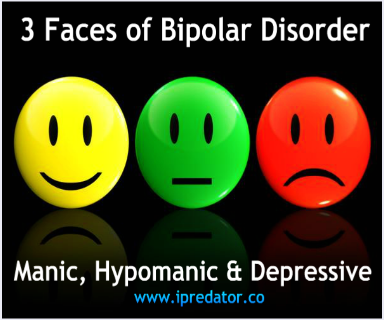
1. When we use Markov-Switching Regression Models
2. Introductory concepts
3. Markov-Switching Dynamic Regression
   - Predictions
     - State probabilities predictions
     - Level predictions
   - State expected durations
   - Transition probabilities
4. Markov-Switching AR Models

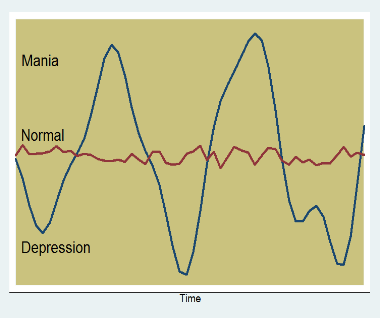
When we use Markov-Switching Regression Models

- The parameters of the data generating process (DGP) vary over a set of different unobserved states.
- We do not know the current state of the DGP, but we can estimate the probability of each possible state.

Markov switching dynamic regression examples

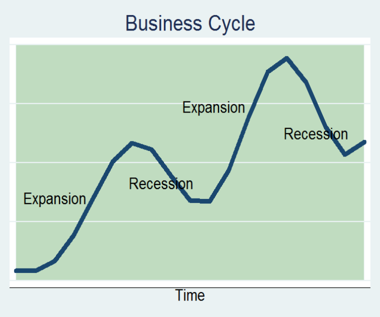
- In Psychology:
  - Manic depressive states (Hamaker et al. 2010).





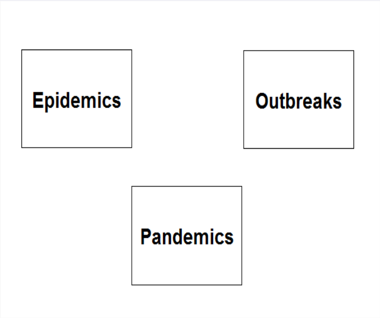
Markov switching dynamic regression examples

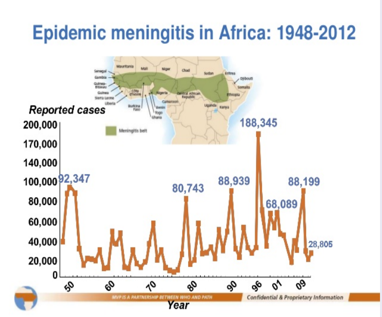
- In Economics:
  - Asymmetrical behavior over GDP expansions and recessions (Hamilton 1989).
  - Exchange rates (Engel and Hamilton 1990).
  - Interest rates (García and Perron 1996).
  - Stock returns (Kim et al. 1998).



Markov switching dynamic regression examples

- In Epidemiology:
  - Incidence rates of infectious disease in epidemic and nonepidemic states (Lu et al. 2010).



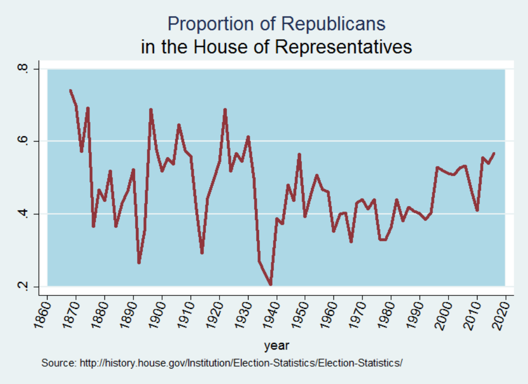


Source:
http://www.slideshare.net/meningitis/1620-mrf-marie-pierre-preziosi-06-nov

Markov switching dynamic regression examples

- In Political Science:
  - Democratic and Republican partisan states in the US congress (Jones et al 2010).

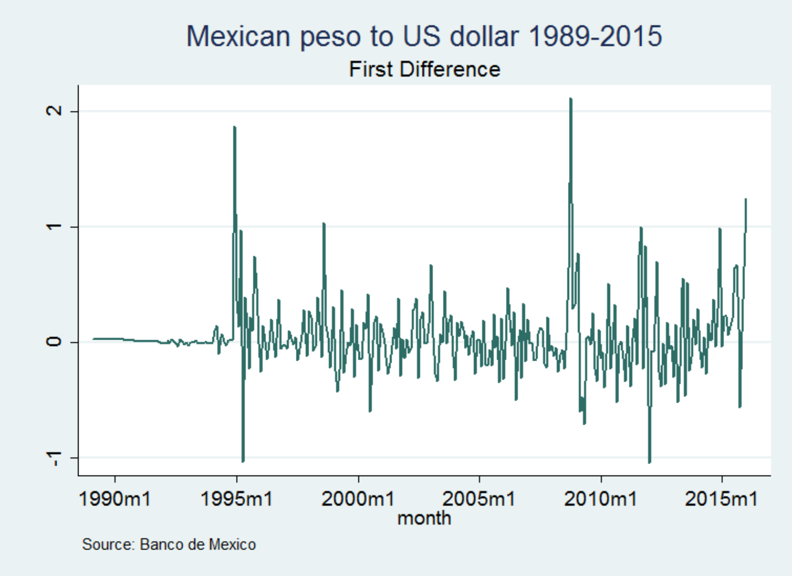
    State 1: Republicans are the dominant national party
    State 2: Democrats are the dominant national party



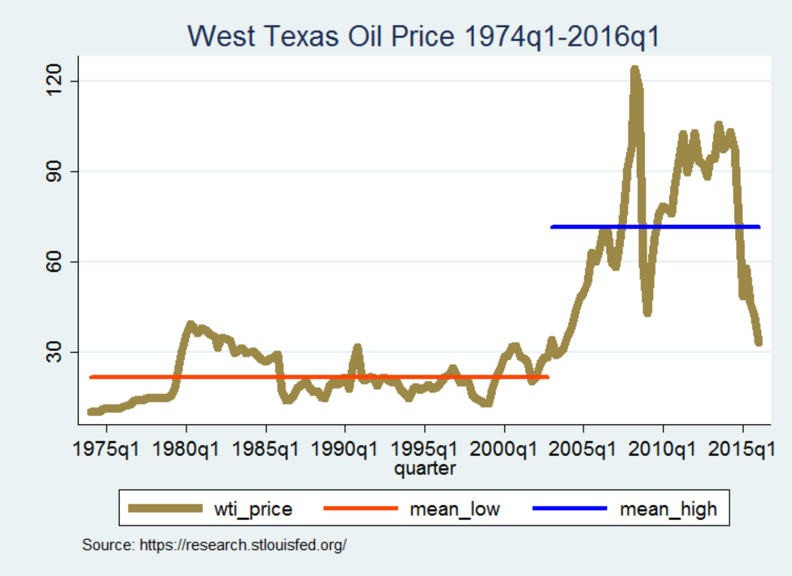
When we use Markov-Switching Regression Models

- The time series in all those examples are characterized by DGPs with dynamics that are state dependent.
  - States may be recessions and expansions, high/low volatility, depressive/non-depressive, epidemic/non-epidemic states, etc.
  - Any of the parameters (beta estimates, sigma, AR components) may be different for each state.

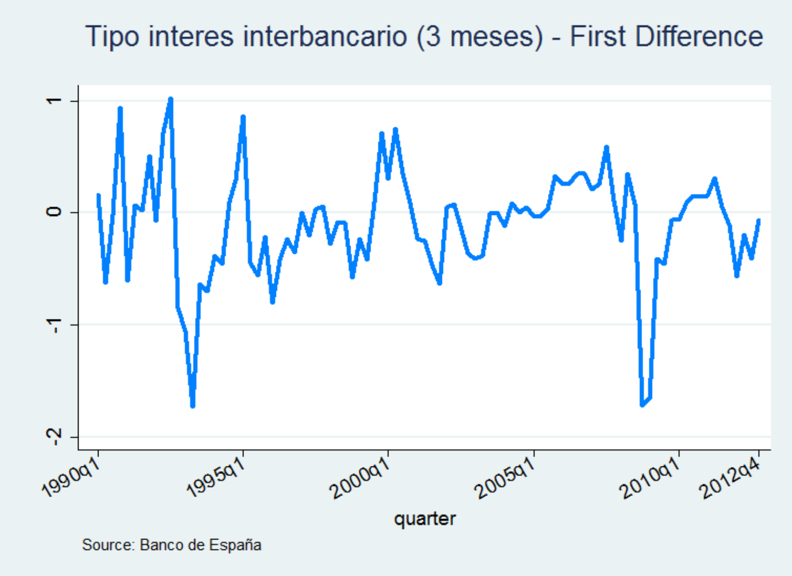
Different volatilities Mexican peso to Us dollar



Different levels, volatilities and slopes- West Texas Oil Price



Different AR structure - Interbank interest rate for Spain



Introductory Concepts

Markov-Switching Regression Models

- Models for time series that transition over a set of finite unobserved states.
- The time of transition between states and the duration in a particular state are both random.
- The transitions follow a Markov process.
- We can estimate state-dependent and state-independent parameters.

Markov-Switching Regression Models

- Let's then define a (first order) Markov Chain:
  - Assume the states are defined by a random variable S_t that takes the integer values 1, 2, ..., N.
  - Then, the probability of the current state, j, only depends on the previous state:

$$P(S_t = j | S_{t-1} = i, S_{t-2} = k, S_{t-3} = w...) = P(S_t = j | S_{t-1} = i) = p_{ij}$$

Markov-Switching Regression Models

- Let's define a simple constant only model with three states:

$$y_t = \mu_{s_t} + \varepsilon_t$$

Where:

$$\begin{aligned} \mu_{s_t} &= \mu_1 & \text{if} \quad s_t = 1 \\ \mu_{s_t} &= \mu_2 & \text{if} \quad s_t = 2 \\ \mu_{s_t} &= \mu_3 & \text{if} \quad s_t = 3 \end{aligned}$$

- We do not know with certainty the current state, but we can estimate the probability of being in each state.
- We can also estimate the transition probabilities:
  - p_{ij} : probability of being in state j in the current period given that the process was in state i in the previous period.

Transition probabilities, expected duration, tests

- We will then be interested in obtaining the matrix with the transition probabilities:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

Where:

$$p_{11} + p_{12} + p_{13} = 1$$
$$p_{21} + p_{22} + p_{23} = 1$$
$$p_{31} + p_{32} + p_{33} = 1$$

- We will also be interested in the expected duration for each state.
- We can perform tests for comparing parameters across states

Markov-switching dynamic regression

Markov-switching dynamic regression

- Allow states to switch according to a Markov process
- Allow for quick adjustments after a change of state.
- Often applied to high frequency data (monthly,weekly,etc.)

Markov-switching dynamic regression

- The model can be written as:

$$y_t = \mu_{s_t} + x_t\alpha + z_t\beta_{s_t} + \epsilon_t$$

Where:

y_t: Dependent variable

x_t: Vector of exog. variables with state invariant coefficients α

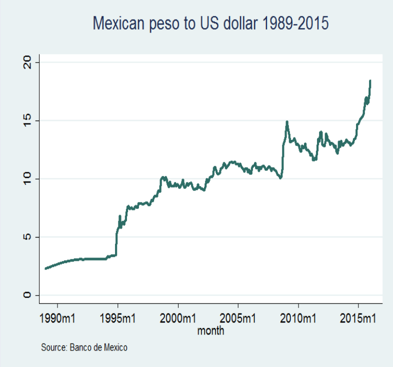
z_t: Vector of exog. variables with state-dependent coefficients β_s

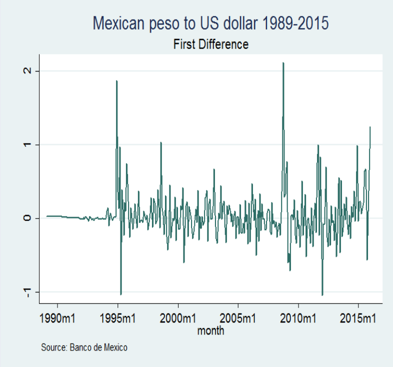
ϵ_{st} ~iid $N(0, \sigma_s^2)$

We can also include lags of the dependent variable among the regressors

Markov switching dynamic regression

- Example 1:
  - Mexican peso to US dollar
  - Period: 1989m1 - 2015m12
  - Source: Banco de México





Markov switching dynamic regression with two states

. mswitch dr D.tc,states(2) varswitch switch(,noconstant) constant nolog

```
Performing EM optimization:
Performing gradient-based optimization:

Markov-switching dynamic regression

Sample:  1989m2 -  2016m1                       No. of obs        =        324
Number of states =    2                         AIC               =    -0.3199
Unconditional probabilities: transition         HQIC              =    -0.2966
                                                SBIC              =    -0.2615
Log likelihood = 56.82268
```

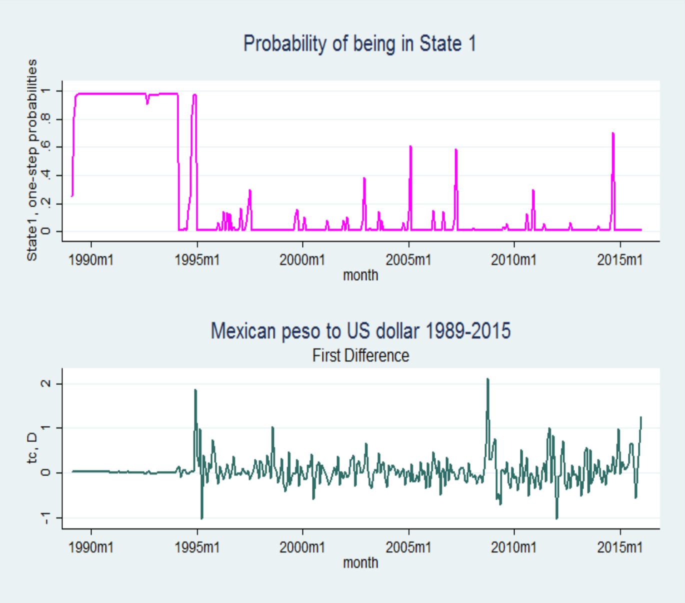
D.tc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.tc						
_cons	.0135642	.0020066	6.76	0.000	.0096313	.0174971
sigma1	.0160312	.0014595			.0134113	.0191628
sigma2	.3603955	.0158708			.330594	.3928835
p11	.9772127	.0196357			.883934	.9958759
p21	.0075981	.0057055			.0017346	.0326345

Markov switching dynamic regression with two states

- Probabilities of being in a given state

```
. predict pr_state1 pr_state2, pr
```

MSDR - Example 1: Probability of being in State 1



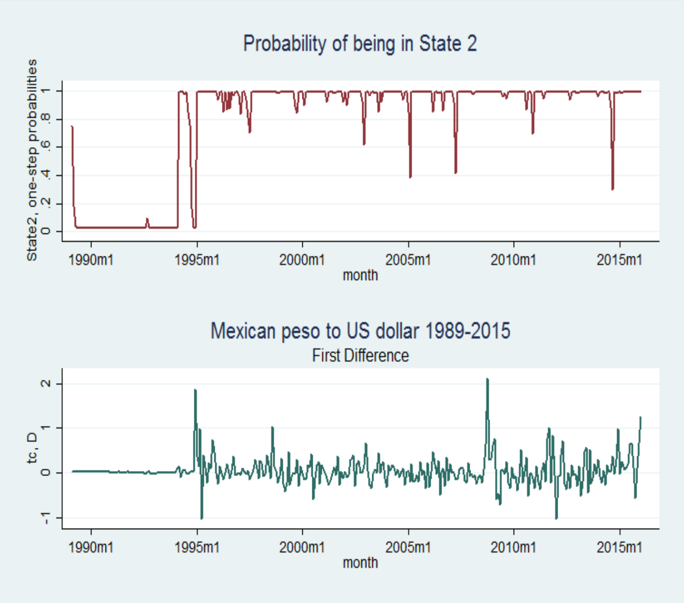
Markov switching dynamic regression with two states

```
Performing EM optimization:
Performing gradient-based optimization:

Markov-switching dynamic regression
Sample:  1989m2 -  2016m1                        No. of obs     =       324
Number of states =    2                          AIC            =   -0.3199
Unconditional probabilities: transition          HQIC           =   -0.2966
                                                 SBIC           =   -0.2615
Log likelihood = 56.82268
```

D.tc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.tc						
_cons	.0135642	.0020066	6.76	0.000	.0096313	.0174971
sigma1	.0160312	.0014595			.0134113	.0191628
sigma2	.3603955	.0158708			.330594	.3928835
p11	.9772127	.0196357			.883934	.9958759
p21	.0075981	.0057055			.0017346	.0326345

MSDR - Example 1: Probability of being in State 2



Markov switching dynamic regression with two states

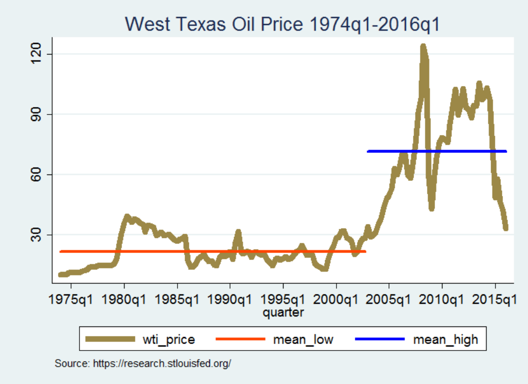
```
Performing EM optimization:
Performing gradient-based optimization:

Markov-switching dynamic regression
Sample:   1989m2 -  2016m1                     No. of obs        =       324
Number of states =     2                       AIC               =   -0.3199
Unconditional probabilities: transition        HQIC              =   -0.2966
                                               SBIC              =   -0.2615
Log likelihood = 56.82268
```

D.tc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.tc						
_cons	.0135642	.0020066	6.76	0.000	.0096313	.0174971
sigma1	.0160312	.0014595			.0134113	.0191628
sigma2	.3603955	.0158708			.330594	.3928835
p11	.9772127	.0196357			.883934	.9958759
p21	.0075981	.0057055			.0017346	.0326345

Markov switching dynamic regression

- Example 2:
  - West Texas Oil Price
  - Period: 1974q1 - 2016q1
  - Source: Federal Reserve Bank of St. Louis



Markov switching dynamic regression for WTI

. mswitch dr wti,varswitch states(2) switch(L(1/2).wti) nolog vsquish

```
Markov-switching dynamic regression

Sample: 1974q3 - 2016q1                         No. of obs      =        167
Number of states =    2                         AIC             =     5.7406
Unconditional probabilities: transition         HQIC            =     5.8163
```

wti	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
State1						
wti						
L1.	1.159938	.1420059	8.17	0.000	.8816118	1.438265
L2.	-.2717729	.1411955	-1.92	0.054	-.548511	.0049652
_cons	7.194071	4.937824	1.46	0.145	-2.483886	16.87203
State2						
wti						
L1.	1.350215	.1098233	12.29	0.000	1.134965	1.565465
L2.	-.3653538	.10544	-3.47	0.001	-.5720124	-.1586951
_cons	.6982948	.5103081	1.37	0.171	-.3018907	1.69848
sigma1	12.33223	1.337368			9.970875	15.25282
sigma2	2.013336	.1763262			1.695777	2.390362
p11	.9061409	.0513983			.7470475	.969287
p21	.0428145	.0223459			.01513	.1152286

Markov switching dynamic regression for WTI

- Test on the equality of intercept across states

```
. test [State1]L1.wti=[State2]L1.wti,notest
( 1)   [State1]L.wti - [State2]L.wti = 0

. test [State1]L2.wti=[State2]L2.wti,accum
( 1)   [State1]L.wti - [State2]L.wti = 0
( 2)   [State1]L2.wti - [State2]L2.wti = 0
          chi2(  2) =    2.61
        Prob > chi2 =    0.2717
```

- Test on the equality of sigma across states

```
. test [lnsigma1]_cons=[lnsigma2]_cons
( 1)   [lnsigma1]_cons - [lnsigma2)]_cons = 0
          chi2(  1) =  199.07
        Prob > chi2 =    0.0000
```

Markov switching dynamic regression for WTI

. mswitch dr wti L(1/2).wti, varswitch states(2) nolog vsquish

```
Markov-switching dynamic regression
Sample: 1974q3 - 2016q1                     No. of obs      =        167
Number of states =     2                    AIC             =     5.7352
Unconditional probabilities: transition     HQIC            =     5.7958
```

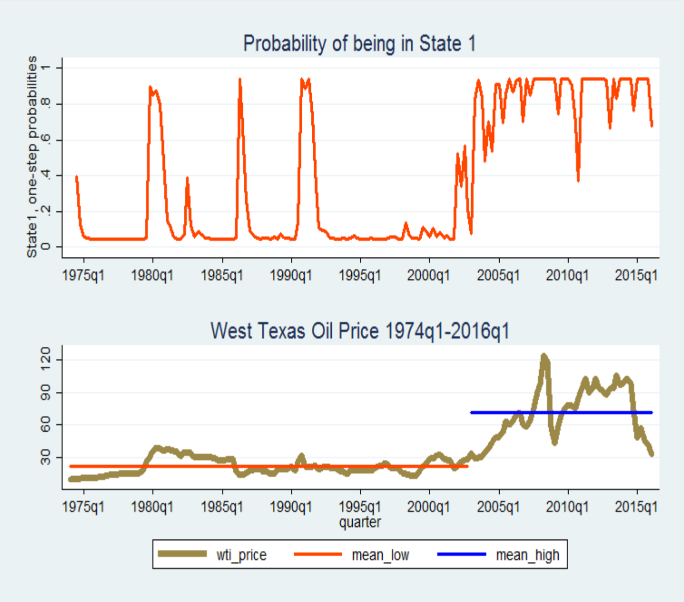
wti	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
wti						
wti						
L1.	1.189054	.1192115	9.97	0.000	.9554034	1.422704
L2.	-.2494644	.1099147	-2.27	0.023	-.4648932	-.0340356
State1						
_cons	3.837488	2.213145	1.73	0.083	-.5001956	8.175171
State2						
_cons	1.441643	.5538878	2.60	0.009	.3560428	2.527243
sigma1	11.06814	1.179045			8.982545	13.63797
sigma2	1.759657	.2833698			1.283374	2.412698
p11	.9394488	.0337386			.8291112	.9802425
p21	.0392075	.022843			.0122803	.1181172

Markov switching dynamic regression

- Predict probabilities of being at each state

predict pr_state1 pr_state2, pr

MSDR - Example 2: Probability of being in State 1



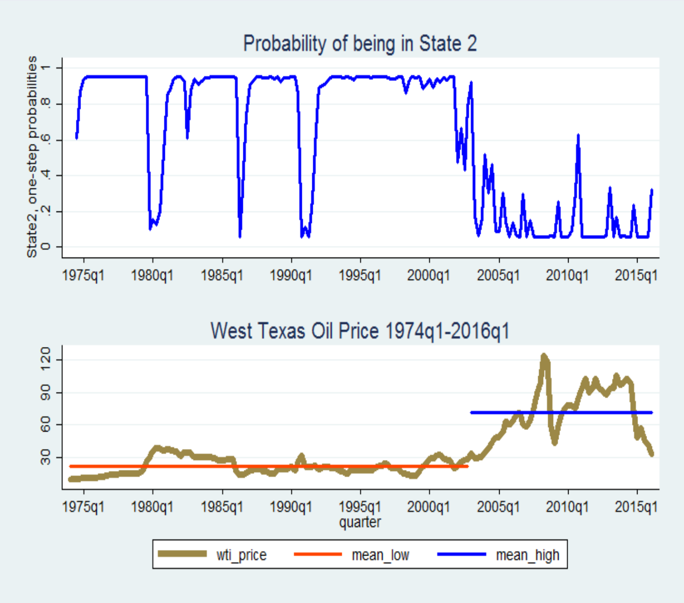
Markov switching dynamic regression for WTI

Markov-switching dynamic regression

Sample: 1974q3 - 2016q1	No. of obs	=	167
Number of states = 2	AIC	=	5.7352
Unconditional probabilities: transition	HQIC	=	5.7958

wti	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
wti						
wti						
L1.	1.189054	.1192115	9.97	0.000	.9554034	1.422704
L2.	-.2494644	.1099147	-2.27	0.023	-.4648932	-.0340356
State1						
_cons	3.837488	2.213145	1.73	0.083	-.5001956	8.175171
State2						
_cons	1.441643	.5538878	2.60	0.009	.3560428	2.527243
sigma1	11.06814	1.179045			8.982545	13.63797
sigma2	1.759657	.2833698			1.283374	2.412698
p11	.9394488	.0337386			.8291112	.9802425
p21	.0392075	.022843			.0122803	.1181172

MSDR - Example 2: Probability of being in State 2



Markov switching dynamic regression for WTI

```
Markov-switching dynamic regression
Sample: 1974q3 - 2016q1                        No. of obs       =        167
Number of states =     2                       AIC              =     5.7352
Unconditional probabilities: transition        HQIC             =     5.7958
```

wti	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
wti						
wti						
L1.	1.189054	.1192115	9.97	0.000	.9554034	1.422704
L2.	-.2494644	.1099147	-2.27	0.023	-.4648932	-.0340356
State1						
_cons	3.837488	2.213145	1.73	0.083	-.5001956	8.175171
State2						
_cons	1.441643	.5538878	2.60	0.009	.3560428	2.527243
sigma1	11.06814	1.179045			8.982545	13.63797
sigma2	1.759657	.2833698			1.283374	2.412698
p11	.9394488	.0337386			.8291112	.9802425
p21	.0392075	.022843			.0122803	.1181172

Markov switching dynamic regression for WTI

- Transition probabilities

  . estat transition

Number of obs = 167

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9394488	.0337386	.8291112	.9802425
p12	.0605512	.0337386	.0197575	.1708888
p21	.0392075	.022843	.0122803	.1181172
p22	.9607925	.022843	.8818828	.9877197

Markov switching dynamic regression for WTI

- Expected duration

  . estat duration

Number of obs = 167

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	16.51496	9.201998	5.851757	50.61379
State2	25.50535	14.85988	8.466169	81.43109

Markov-switching AR model

Markov-switching AR model

- Allow states to switch according to a Markov process
- Allow a gradual adjustment after a change of state.
- Often applied to lower frequency data (quarterly, yearly, etc.)

Markov-switching AR model

- The model can be written as:

$$y_t = \mu_{s_t} + x_t\alpha + z_t\beta_{s_t} + \sum_{i=1}^{P} \phi_{i,s_t}(y_{t-i} - \mu_{s_{t-i}} - x_{t-i}\alpha + z_{t-i}\beta_{s_{t-i}}) + \epsilon_{t,s_t}$$

Where:

y_t: Dependent variable

μ_{s_t}: State-dependent intercept

x_t: Vector of exog. variables with state invariant coefficients α

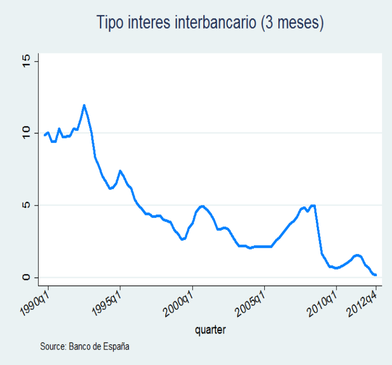
z_t: Vector of exog. variables with state-dependent coefficients β_{s_t}

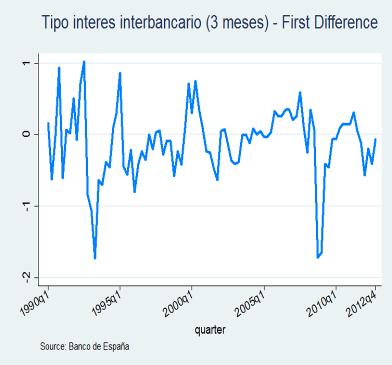
ϕ_{i,s_t}: i^{th} AR term in state s_t

ϵ_{t,s_t} ~iid $N(0, \sigma_s^2)$

Markov switching AR model

- Example 3:
  - Interbank interest rate for Spain
  - Period: 1989Q4 - 2015Q3
  - Source: Banco de España





Markov switching AR model

```
. mswitch ar D.r_interbank D.ipc,states(2) ar(1) ///
    arswitch varswitch switch(,noconstant) constant
```

```
Markov-switching autoregression
Sample: 1990q2 - 2012q4                      No. of obs     =        91
Number of states =    2                      AIC            =    1.1681
Unconditional probabilities: transition      HQIC           =    1.2572
```

D. r_interbank	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.r_interb~k						
ipc						
D1.	.1345492	.0430415	3.13	0.002	.0501895	.218909
_cons	-.1287786	.0299325	-4.30	0.000	-.1874453	-.0701119
State1						
ar						
L1.	-.5821326	.0868487	-6.70	0.000	-.7523529	-.4119122
State2						
ar						
L1.	.600846	.1133802	5.30	0.000	.3786249	.8230671
sigma1	.10039	.021533			.0659346	.1528509
sigma2	.4279839	.0404373			.3556339	.5150526

Markov switching AR model

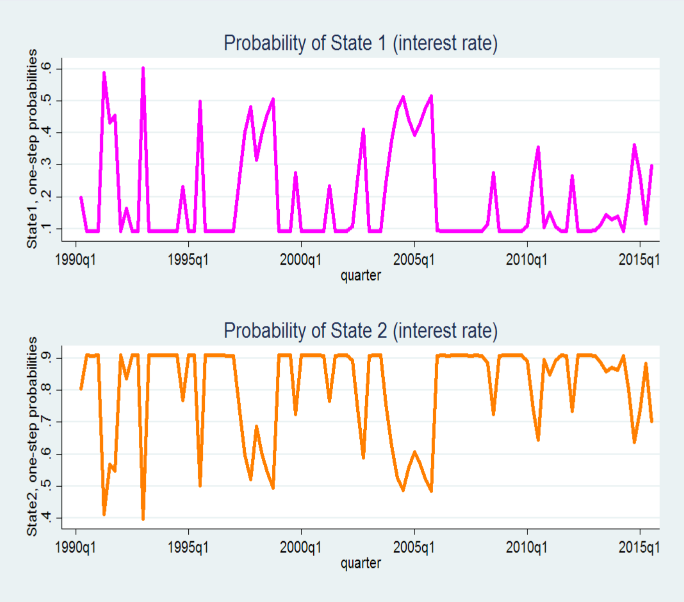
```
. estat transition
Number of obs = 91
```

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.6238106	.1906249	.2523082	.8906938
p12	.3761894	.1906249	.1093062	.7476918
p21	.0917497	.0529781	.0282364	.2599153
p22	.9082503	.0529781	.7400847	.9717636

```
. estat duration
Number of obs = 91
```

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	2.658235	1.346997	1.33745	9.148609
State2	10.89922	6.293423	3.847408	35.41533

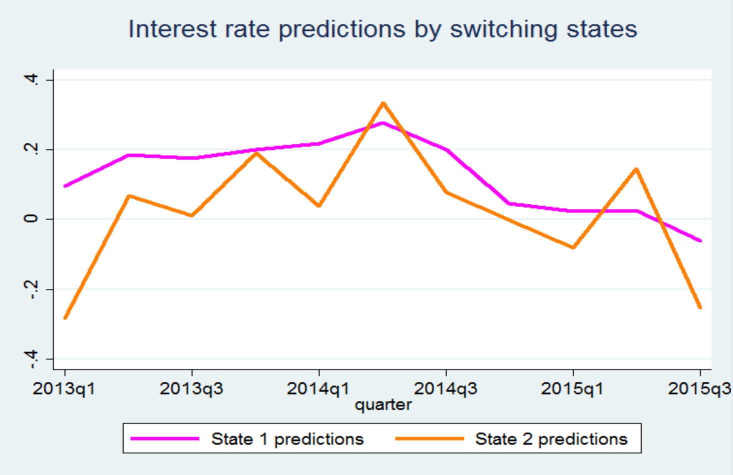
MSAR - Example 3: Probability of being in each State



Markov switching AR model

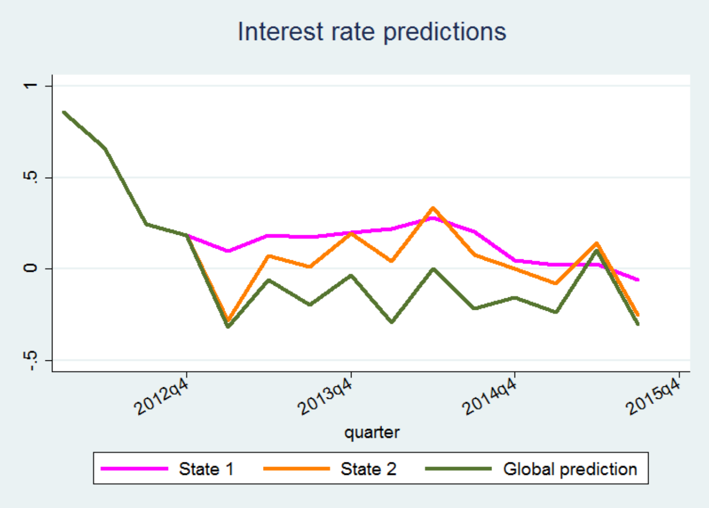
```
. predict state*,yhat dynamic(tq(2012q4))

. forvalues i=1/2 {
  2.    generate y_st`i´=state`i´+L.r_interbank
  3. }
```



Markov switching AR model

. predict r_hat,yhat dynamic(tq(2012q4))



Summary

1. When we use Markov-Switching Regression Models
2. Introductory concepts
3. Markov-Switching Dynamic Regression
    - Predictions
        - State probabilities predictions
        - Level predictions
    - State expected durations
    - Transition probabilities
4. Markov-Switching AR Models

References

- Engel, C., and J. D. Hamilton. 1990. *Long swings in the dollar: Are they in the data and do markets know it?*. American Economic Review 80: 689—713.
- Hamilton, J. D. 1989. *A new approach to the economic analysis of nonstationary time series and the business cycle*. Econometrica 57: 357—384.
- Garcia, R., and P. Perron. 1996. *An analysis of the real interest rate under regime shifts*. Review of Economics and Statistics 78: 111—125.
- Kim, C.-J., C. R. Nelson, and R. Startz. 1998. *Testing for mean reversion in heteroskedastic data based on Gibbs-sampling-augmented randomization*. Journal of Empirical Finance 5: 115—43.
- Lu, H.-M., D. Zeng, and H. Chen. 2010. *Prospective infectious disease outbreak detection using Markov switching models*. IEEE Transactions on Knowledge and Data Engineering 22: 565—577.
- Hamaker, E. L., R. P. P. P. Grasman, and J. H. Kamphuis. 2010. *Regime-switching models to study psychological processes*. In Individual Pathways of Change: Statistical Models for Analyzing Learning and Development, ed. P. C. Molenaar and K. M. Newell, 155—168. Washington, DC: American Psychological Association