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procrustes postestimation —	<ul> <li>Postestimation</li> </ul>	tools	for procrustes
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# **Postestimation commands**

The following postestimation commands are of special interest after procrustes:

Command	Description
estat compare	fit statistics for orthogonal, oblique, and unrestricted transformations
estat mvreg	display multivariate regression resembling unrestricted transformation
estat summarize	display summary statistics over the estimation sample
procoverlay	produce a Procrustes overlay graph

The following standard postestimation commands are also available:

Command	Description
*estimates	cataloging estimation results
predict	predictions and residuals

 $<sup>^{*}</sup>$ All estimates subcommands except table and stats are available; see [R] estimates.

## predict

### **Description for predict**

predict creates new variables containing predictions such as fitted values, unstandardized residuals, and residual sum of squares.

### Menu for predict

Statistics > Postestimation

#### Syntax for predict

predict	$[type] \{stub*   newvarlist\} [if] [in], [statistic]$
statistic	Description
Main	
<u>fit</u> ted	fitted values 1 c' + $\rho$ X A; the default (specify $\#_y$ vars)
<u>res</u> iduals	unstandardized residuals (specify $\#_y$ vars)
q	residual sum of squares over the target variables (specify one var)

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## **Options for predict**

Main

fitted, the default, computes fitted values, that is, the least-squares approximations of the target (varlisty) variables. You must specify the same number of new variables as there are target variables.

residuals computes the raw (unstandardized) residuals for each target  $(varlist_y)$  variable. You must specify the same number of new variables as there are target variables.

q computes the residual sum of squares over all variables, that is, the squared Euclidean distance between the target and transformed source points. Specify one new variable.

#### estat

### **Description for estat**

estat compare displays a table with fit statistics of the three transformations provided by procrustes: orthogonal, oblique, and unrestricted. The two additional procrustes analyses are performed on the same sample as the original procrustes analysis and with the same options. Ftests comparing the models are provided.

estat mvreg produces the mvreg (see [MV] mvreg) output related to the unrestricted Procrustes analysis (the transform(unrestricted) option of procrustes).

estat summarize displays summary statistics over the estimation sample of the target and source variables ( $varlist_y$  and  $varlist_x$ ).

#### Menu for estat

Statistics > Postestimation

#### Syntax for estat

```
Table of fit statistics
  estat compare [, detail]
```

Comparison of mvreg and procrustes output

```
estat mvreg _options
```

Display summary statistics

```
estat <u>summarize</u> [, <u>lab</u>els <u>nohea</u>der <u>noweights</u>]
```

collect is allowed with estat compare and estat summarize; see [U] 11.1.10 Prefix commands.

## Options for estat

detail, an option with estat compare, displays the standard procrustes output for the two additional transformations.

mvreg\_options, allowed with estat mvreg, are any of the options allowed by mvreg; see [MV] mvreg. The constant is already suppressed if the Procrustes analysis suppressed it.

labels, noheader, and noweights are the same as for the generic estat summarize command; see [R] estat summarize.

# procoverlay

## **Description for procoverlay**

procoverlay displays a plot of the target variables overlaid with the fitted values derived from the source variables. If there are more than two target variables, multiple plots are shown in one graph.

### Menu for procoverlay

Statistics > Multivariate analysis > Procrustes overlay graph

# Syntax for procoverlay

$\verb"procoverlay" \left[ \textit{if} \right] \left[ \textit{in} \right]$	[, procoverlay_options]
procoverlay_options	Description
Main	
<u>auto</u> aspect	adjust aspect ratio on the basis of the data; default aspect ratio is 1
<pre>targetopts(target_opts)</pre>	affect the rendition of the target
<pre>source opts(source_opts)</pre>	affect the rendition of the source
Y axis, X axis, Titles, Legend, Overall	
twoway_options	any options other than by () documented in [G-3] twoway_options
Ву	
<pre>byopts(by_option)</pre>	affect the rendition of combined graphs
target_opts	Description
Main	
<u>nolab</u> el	removes the default observation label from the target
marker_options	change look of markers (color, size, etc.)
marker_label_options	change look or position of marker labels
source_opts	Description
Main	
<u>nolab</u> el	removes the default observation label from the source
marker_options	change look of markers (color, size, etc.)
marker_label_options	change look or position of marker labels

#### Options for procoverlay

Main

autoaspect specifies that the aspect ratio be automatically adjusted based on the range of the data to be plotted. This option can make some procoverlay plots more readable. By default, procoverlay uses an aspect ratio of one, producing a square plot.

As an alternative to autoaspect, the  $twoway\_option$  aspectratio() can be used to override the default aspect ratio. procoverlay accepts the aspectratio() option as a suggestion only and will override it when necessary to produce plots with balanced axes, that is, where distance on the x axis equals distance on the y axis.

twoway\_options, such as xlabel(), xscale(), ylabel(), and yscale(), should be used with caution. These axis\_options are accepted but may have unintended side effects on the aspect ratio. See [G-3] twoway\_options.

targetopts(target\_opts) affects the rendition of the target plot. The following target\_opts are allowed:

nolabel removes the default target observation label from the graph.

marker\_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker\_options.

marker\_label\_options specify if and how the markers are to be labeled; see [G-3] marker\_label\_options.

sourceopts(source\_opts) affects the rendition of the source plot. The following source\_opts are allowed:

nolabel removes the default source observation label from the graph.

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

marker\_label\_options specify if and how the markers are to be labeled; see [G-3] marker\_label\_options.

Y axis, X axis, Titles, Legend, Overall

twoway\_options are any of the options documented in [G-3] twoway\_options, excluding by(). These
include options for titling the graph (see [G-3] title\_options) and for saving the graph to disk
(see [G-3] saving\_option). See autoaspect above for a warning against using options such as
xlabel(), xscale(), ylabel(), and yscale().

Ву

byopts(*by\_option*) is documented in [G-3] *by\_option*. This option affects the appearance of the combined graph and is ignored, unless there are more than two target variables specified in procrustes.

## Remarks and examples

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The examples in [MV] **procrustes** demonstrated a Procrustes transformation of a historical map, produced by John Speed in 1610, to a modern map. Here we demonstrate the use of procrustes postestimation tools in assessing the accuracy of Speed's map. Example 1 of [MV] **procrustes** performed the following analysis:

```
. use https://www.stata-press.com/data/r18/speed_survey
(Data on Speed's Worcestershire map (1610))
```

. procrustes (survey\_x survey\_y) (speed\_x speed\_y)
 (output omitted)

See example 1 of [MV] procrustes. The following examples are based on this procrustes analysis.

#### Example 1: Predictions

Did John Speed get the coordinates of the towns right—up to the location, scale, and orientation of his map relative to the modern map? In example 1 of [MV] **procrustes**, we demonstrated how the optimal transformation from the historical coordinates to the modern (true) coordinates can be estimated by procrustes.

It is possible to "predict" the configuration of 20 cities on Speed's historical map, optimally transformed (rotated, dilated, and translated) to approximate the true configuration. predict with the fitted option expects the same number of variables as the number of target (dependent) variables (survey\_x and survey\_y).

```
. predict fitted_x fitted_y
(option fitted assumed; fitted values)
```

We omitted the fitted option because it is the default.

It is often useful to also compute the (squared) distance between the true location and the transformed location of the historical map. This can be seen as a quality measure—the larger the value, the more Speed erred in the location of the respective town.

. predict q, q

We now list the target data (survey\_x and survey\_y, the values from the modern map), the fitted values (fitted\_x and fitted\_y, produced by predict), and the squared distance between them (q, produced by predict with the q option).

liet	nama	SIITVEV	v	CHETTAT	77	fitted	v	fitted	77	α.	can(n)	noohe

name	survey_x	survey_y	fitted_x	fitted_y	q
Alve	1027	725	1037.117	702.9464	588.7149
Arro	1083	565	1071.682	562.6791	133.4802
Astl	787	677	783.0652	674.5216	21.62482
Beck	976	358	978.8665	366.3761	78.37637
Beng	1045	435	1055.245	431.6015	116.51
Crad	736	471	725.8594	476.5895	134.075
Droi	893	633	890.5839	633.6066	6.205747
Ecki	922	414	929.4932	411.1757	64.12465
Eves	1037	437	1036.887	449.2707	150.5827
Hall	828	579	825.1494	575.9836	17.22464
Hanb	944	637	954.6189	643.6107	156.4629
Inkb	1016	573	1004.869	577.1111	140.7917
Kemp	848	490	845.7215	490.8959	5.994327
Kidd	826	762	836.8665	760.5699	120.1264
Mart	756	598	745.2623	597.5585	115.4937
Stud	1074	632	1072.622	634.3164	7.264294
Tewk	891	324	898.4571	318.632	84.42448
UpSn	943	544	939.3932	545.8247	16.33858
Upto	852	403	853.449	400.9419	6.335171
Worc	850	545	848.7917	547.7881	9.233305

We see that Speed especially erred in the location of Alvechurch—it is off by no less than  $\sqrt{588} = 24$  miles, whereas the average error is about 8 miles. In a serious analysis of this dataset, we would check the data on Alvechurch, and, if we found it to be in order, consider whether we should actually drop Alvechurch from the analysis. In this illustration, we ignore this potential problem.

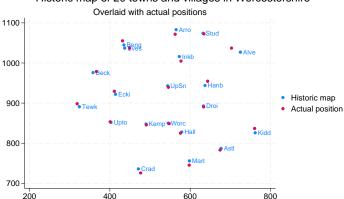
### Example 2: Procrustes overlay graph

Although the numerical information convinces us that Speed's map is generally accurate, a plot will convey this message more convincingly. procoverlay produces a plot that contains the target (survey) coordinates and the Procrustes-transformed historical coordinates. We could just type

. procoverlay

However, we decide to set several options to produce a presentation-quality graph. The suboption mlabel() of target() (or of source()) adds labels, identifying the towns. Because the target and source points are so close, there can be no confusing how they are matched. Displaying the labels twice in the plot is not helpful for this dataset. Therefore, we choose to label the target points, but not the source points using the nolabel suboption of source(). We preserve the equivalence of the x and y scale while using as much of the graphing region as possible with the autoaspect option. The span suboption of title() allows the long title to extend beyond the graph region if needed. We override the default legend by using the legend() option.

- . procoverlay, target(mlabel(name)) source(nolabel) autoaspect
- > title(Historic map of 20 towns and villages in Worcestershire, span)
- > subtitle(Overlaid with actual positions)
- > legend(label(1 Historic map) label(2 Actual position))



#### Historic map of 20 towns and villages in Worcestershire

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## Example 3: estat

estat offers three specific facilities after procrustes. These can all be seen as convenience tools that accomplish simple analyses, ensuring that the same variables and the same observations are used as in the Procrustes analysis.

The variables involved in the Procrustes analysis can be summarized over the estimation sample, for instance, to gauge differences in scales and location of the target and source variables.

. estat summarize

Estimation sam	mple procrustes	Number of obs = 20					
Variable	Mean	Std. dev.	Min	Max			
target survey_x survey_y	916.7 540.1	106.6993 121.1262	736 324	1083 762			
source speed_x speed_y	153.95 133.9	46.76084 49.90401	78 40	220 220			

From the summarization, the two maps have different origins and scale.

As pointed out in [MV] procrustes, orthogonal and oblique Procrustes analyses can be thought of as special cases of multivariate regression (see [MV] mvreg), subject to nonlinear restrictions on the coefficient matrix. Comparing the Procrustes statistics and the transformations for each of the three classes of transformations is helpful in selecting a transformation. The compare subcommand of estat provides summary information for the optimal transformations in each of the three classes.

. estat compare

Summary statistics for three transformations

	Procrustes	df_m	df_r	rmse
orthogonal oblique unrestricted	0.0040	4	36	7.403797
	0.0040	5	35	7.498294
	0.0037	6	34	7.343334

(F tests comparing the models suppressed)

The Procrustes statistic is ensured to decrease (not increase) from orthogonal to oblique to unrestricted because the associated classes of transformations are getting less restrictive. The model degrees of freedom (df\_m) of the three transformation classes are the dimension of the classes, that is, the number of "free parameters". For instance, with orthogonal transformations between two source and two target variables, there is 1 degree of freedom for the rotation (representing the rotation angle), 2 degrees of freedom for the translation, and 1 degree of freedom for dilation (uniform scaling), that is, four in total. The residual degrees of freedom  $(df_r)$  are the number of observations (number of target variables times the number of observations) minus the model degrees of freedom. The root mean squared error RMSE, defined as

$$\mathrm{RMSE} = \sqrt{\frac{\mathrm{RSS}}{\mathrm{df}_r}}$$

does not, unlike the Procrustes statistic, surely become smaller with the less restrictive models. In this example, in fact, the RMSE of the orthogonal transformation is smaller than that of the oblique transformation. This indicates that the additional degree of freedom allowing for skew rotations does not produce a closer fit. In this example, we see little reason to relax orthogonal transformations; very little is gained in terms of the Procrustes statistic (an illness-of-fit measure) or the RMSE. In this interpretation, we used our intuition to guide us whether a difference in fit is substantively and statistically meaningful—formal significance tests are not provided.

Finally, the unrestricted transformation can be estimated with procrustes ..., transform(unrestricted). This analysis is related to a multivariate regression with the target variables as the dependent variables and the source variables as the independent variables. Although the unrestricted Procrustes analysis assumes spherical (uncorrelated homoskedastic) residuals, this

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restrictive assumption is not made in multivariate regression as estimated by the mvreg command. The comparable multivariate regression over the same estimation sample can be viewed simply by typing

. estat mvreg

${\tt Multivariate}$	regression,	similar	to "	procrustes	, tı	cansform(unre	stricted)"
Equation	Obs	Parms		RMSE	"R-sq"	F	P>F
survey_x	20	3	7	7.696981	0.9953	1817.102	0.0000
survey_y	20	3	6	3.971772	0.9970	2859.068	0.0000
	Coefficier	nt Std.	err.	t	P> t	[95% conf	. interval]
survey_x							
speed_x	2.27584	.0379	9369	59.99	0.000	2.1958	2.35588
speed_y	.4147244	.035	5475	11.67	0.000	.3397257	.489723
_cons	510.8028	8.06	5519	63.33	0.000	493.7861	527.8196
survey_y							
speed_x	4129564	.0343	3625	-12.02	0.000	485455	3404579
speed_y	2.355725	.032	1982	73.16	0.000	2.287793	2.423658
_cons	288.243	7.30	5587	39.46	0.000	272.8296	303.6564

This analysis is seen as postestimation after a Procrustes analysis, so it does not change the "last estimation results". We may still replay procrustes and use other procrustes postestimation commands.

### Stored results

estat compare after procrustes stores the following in r():

Matrices

r(cstat) Procrustes statistics, degrees of freedom, and RMSEs

F statistics, degrees of freedom, and p-values r(fstat)

estat mvreg does not return results.

estat summarize after procrustes stores the following in r():

Matrices

r(stats) means, standard deviations, minimums, and maximums

### Methods and formulas

The predicted values for the jth variable are defined as

$$\widehat{y}_j = \widehat{c}_j + \widehat{\rho} \mathbf{X} \ \widehat{\mathbf{A}}[.,j]$$

The residual for  $y_j$  is simply  $y_j - \hat{y}_j$ . The "rowwise" quality q of the approximation is defined as the residual sum of squares:

$$q = \sum_{j} (y_j - \widehat{y}_j)^2$$

The entries of the summary table produced by estat compare are described in Methods and formulas of [MV] **procrustes**. The F tests produced by estat compare are similar to standard nested model tests in linear models.

#### References

See References in [MV] procrustes.

#### Also see

[MV] **procrustes** — Procrustes transformation

[MV] **mvreg** — Multivariate regression

[U] 20 Estimation and postestimation commands

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