# Title

mixed postestimation — Postestimation tools for mixed

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# **Postestimation commands**

The following postestimation commands are of special interest after mixed:

Command	Description
estat df	calculate and display degrees of freedom for fixed effects
estat group	summarize the composition of the nested groups
estat icc	estimate intraclass correlations
estat recovariance	display the estimated random-effects covariance matrices
estat sd	display variance components as standard deviations and correlations
estat wcorrelation	display within-cluster correlations and standard deviations

The following standard postestimation commands are also available:

Command	Description			
contrast	contrasts and ANOVA-style joint tests of estimates			
estat ic	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC)			
estat summarize	summary statistics for the estimation sample			
estat vce	variance-covariance matrix of the estimators (VCE)			
estimates	cataloging estimation results			
etable	table of estimation results			
hausman	Hausman's specification test			
lincom	point estimates, standard errors, testing, and inference for linear combina- tions of coefficients			
lrtest	likelihood-ratio test			
margins	marginal means, predictive margins, marginal effects, and average marginal effects			
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)			
nlcom	point estimates, standard errors, testing, and inference for nonlinear com- binations of coefficients			
predict	predictions and their SEs, residuals, etc.			
predictnl	point estimates, standard errors, testing, and inference for generalized predictions			

pwcompare	pairwise comparisons of estimates
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

# predict

### Description for predict

predict creates a new variable containing predictions such as linear predictions, standard errors, fitted values, residuals, and standardized residuals.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

Syntax for obtaining predictions of the outcome and other statistics

predict [type] newvar [if] [in] [, statistic relevel(levelvar)]

Syntax for obtaining BLUPs of random effects and the BLUPs' standard errors

```
predict [type] { stub* | newvarlist } [if] [in], reffects [relevel(levelvar)
reses(stub* | newvarlist)]
```

Syntax for obtaining scores after ML estimation

predict [type] stub\* [if] [in], scores

statistic	Description
Main	
xb	linear prediction for the fixed portion of the model only; the default
stdp	standard error of the fixed-portion linear prediction
<u>fit</u> ted	fitted values, fixed-portion linear prediction plus contributions based on predicted random effects
<u>res</u> iduals * <u>rsta</u> ndard	residuals, response minus fitted values standardized residuals

Unstarred statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.

## **Options for predict**

Main

xb, the default, calculates the linear prediction  $\mathbf{x}\beta$  based on the estimated fixed effects (coefficients) in the model. This is equivalent to fixing all random effects in the model to their theoretical mean value of 0.

stdp calculates the standard error of the linear predictor  $\mathbf{x}\boldsymbol{\beta}$ .

fitted calculates fitted values, which are equal to the fixed-portion linear predictor *plus* contributions based on predicted random effects, or in mixed-model notation,  $x\beta$ +Zu. By default, the fitted values take into account random effects from all levels in the model; however, if the relevel(*levelvar*) option is specified, then the fitted values are fit beginning with the topmost level down to and including level *levelvar*. For example, if classes are nested within schools, then typing

. predict yhat\_school, fitted relevel(school)

would produce school-level predictions. That is, the predictions would incorporate school-specific random effects but not those for each class nested within each school.

- residuals calculates residuals, equal to the responses minus fitted values. By default, the fitted values take into account random effects from all levels in the model; however, if the relevel(*levelvar*) option is specified, then the fitted values are fit beginning at the topmost level down to and including level *levelvar*.
- rstandard calculates standardized residuals, equal to the residuals multiplied by the inverse square root of the estimated error covariance matrix.
- reffects calculates best linear unbiased predictions (BLUPs) of the random effects. By default, BLUPs
  for all random effects in the model are calculated. However, if the relevel(levelvar) option is
  specified, then BLUPs for only level levelvar in the model are calculated. For example, if classes
  are nested within schools, then typing

. predict b\*, reffects relevel(school)

would produce BLUPs at the school level. You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub\** and let Stata name the variables *stub1*, *stub2*, ..., *stubq* for you.

Rabe-Hesketh and Skrondal (2022, sec. 2.11.2) discuss the link between the empirical Bayes predictions and BLUPs and how these predictions are unbiased. They are unbiased when the groups associated with the random effects are expected to vary in repeated samples. If you expect the groups to be fixed in repeated samples, then these predictions are no longer unbiased.

scores calculates the parameter-level scores, one for each parameter in the model including regression coefficients and variance components. The score for a parameter is the first derivative of the log likelihood (or log pseudolikelihood) with respect to that parameter. One score per highest-level group is calculated, and it is placed on the last record within that group. Scores are calculated in the estimation metric as stored in e(b).

scores is not available after restricted maximum-likelihood (REML) estimation.

- relevel (*levelvar*) specifies the level in the model at which predictions involving random effects are to be obtained; see the options above for the specifics. *levelvar* is the name of the model level and is either the name of the variable describing the grouping at that level or is \_all, a special designation for a group comprising all the estimation data.
- reses(*stub*\* | *newvarlist*) calculates the standard errors of the BLUPs of the random effects. By default, standard errors for all BLUPs in the model are calculated. However, if the relevel(*levelvar*)

option is specified, then standard errors for only level *levelvar* in the model are calculated; see the reffects option.

You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub*\* and let Stata name the variables *stub*1, *stub*2, ..., *stubq* for you. The new variables will have the same storage type as the corresponding random-effects variables.

The reffects and reses() options often generate multiple new variables at once. When this occurs, the random effects (or standard errors) contained in the generated variables correspond to the order in which the variance components are listed in the output of mixed. Still, examining the variable labels of the generated variables (with the describe command, for instance) can be useful in deciphering which variables correspond to which terms in the model.

## margins

## **Description for margins**

margins estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

margins	[marginlist]	, options		
margins	[marginlist]	predict(statistic	)	[options]

statistic	Description
xb	linear predictor for the fixed portion of the model only; the default
stdp	not allowed with margins
<u>fit</u> ted	not allowed with margins
<u>res</u> iduals	not allowed with margins
<u>rsta</u> ndard	not allowed with margins
<u>ref</u> fects	not allowed with margins
<u>sc</u> ores	not allowed with margins

Statistics not allowed with margins are functions of stochastic quantities other than e(b). For the full syntax, see [R] margins.

## test and testparm

### Description for test and testparm

test and testparm, by default, perform  $\chi^2$  tests of simple and composite linear hypotheses about the parameters for the most recently fit mixed model. They also support F tests with a small-sample adjustment for fixed effects.

### Menu for test and testparm

Statistics > Postestimation

### Syntax for test and testparm

```
test (spec) [(spec) ...] [, test_options small]
```

```
testparm varlist[, testparm_options small]
```

### Options for test and testparm

Options

- *testparm\_options*; see options of testparm in [R] **test**. Options df() and nosvyadjust may not be specified together with small.
- small specifies that F tests for fixed effects be carried out with the denominator degrees of freedom (DDF) obtained by the same method used in the most recently fit mixed model. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the DDF for some tests may not be available. See Small-sample inference for fixed effects in [ME] mixed for more details.

*test\_options*; see [R] **test** options. Options df(), common, and nosvyadjust may not be specified together with small.

## lincom

#### **Description for lincom**

lincom, by default, computes point estimates, standard errors, z statistics, p-values, and confidence intervals for linear combinations of coefficients after mixed. lincom also provides t statistics for linear combinations of the fixed effects, with the degrees of freedom calculated by the DF method specified in option dfmethod() of mixed.

### Menu for lincom

Statistics > Postestimation

#### Syntax for lincom

lincom exp [, lincom\_options small]

### Options for lincom

*lincom\_options*; see [R] **lincom** options. Option df() may not be specified together with small.

small specifies that t statistics for linear combinations of fixed effects be displayed with the degrees
of freedom obtained by the same method used in the most recently fit mixed model. If option
dfmethod() is not specified in the previous mixed command, option small is not allowed. For
certain methods, the degrees of freedom for some linear combinations may not be available. See
Small-sample inference for fixed effects in [ME] mixed for more details.

## contrast

### **Description for contrast**

contrast, by default, performs  $\chi^2$  tests of linear hypotheses and forms contrasts involving factor variables and their interactions for the most recently fit mixed model. contrast also supports tests with small-sample adjustments after mixed, dfmethod().

#### Menu for contrast

Statistics > Postestimation

### Syntax for contrast

contrast termlist [, contrast\_options small]

### **Options for contrast**

- *contrast\_options*; see [R] **contrast** options. Options df() and nosvyadjust may not be specified together with small.
- small specifies that tests for contrasts be carried out with the DDF obtained by the same method used in the most recently fit mixed model. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the DDF for some contrasts may not be available. See Small-sample inference for fixed effects in [ME] mixed for more details.

#### pwcompare

#### **Description for pwcompare**

pwcompare performs pairwise comparisons across the levels of factor variables from the most recently fit mixed model. pwcompare, by default, reports the comparisons as contrasts (differences) of margins along with z tests or confidence intervals for the pairwise comparisons. pwcompare also supports t tests with small-sample adjustments after mixed, dfmethod().

#### Menu for pwcompare

Statistics > Postestimation

#### Syntax for pwcompare

pwcompare marginlist [, pwcompare\_options small]

#### Options for pwcompare

*pwcompare\_options*; see [R] **pwcompare** options. Option df() may not be specified together with small.

small specifies that t tests for pairwise comparisons be carried out with the degrees of freedom
obtained by the same method used in the most recently fit mixed model with the dfmethod()
option. If option dfmethod() is not specified in the previous mixed command, option small is
not allowed. For certain methods, the degrees of freedom for some pairwise comparisons may not
be available. See Small-sample inference for fixed effects in [ME] mixed for more details.

## **Remarks and examples**

stata.com

Various predictions, statistics, and diagnostic measures are available after fitting a mixed model using mixed. For the most part, calculation centers around obtaining BLUPs of the random effects. Random effects are not estimated when the model is fit but instead need to be predicted after estimation. Calculation of intraclass correlations, estimating the dependence between responses for different levels of nesting, may also be of interest.

#### Example 1: Obtaining predictions of random effects and checking model fit

In example 3 of [ME] mixed, we modeled the weights of 48 pigs measured on nine successive weeks as

$$\texttt{weight}_{ij} = \beta_0 + \beta_1 \texttt{week}_{ij} + u_{0j} + u_{1j} \texttt{week}_{ij} + \epsilon_{ij} \tag{1}$$

for i = 1, ..., 9, j = 1, ..., 48,  $\epsilon_{ij} \sim N(0, \sigma_{\epsilon}^2)$ , and  $u_{0j}$  and  $u_{1j}$  normally distributed with mean 0 and variance–covariance matrix

$$\mathbf{\Sigma} = \operatorname{Var} \begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} \sigma_{u0}^2 & \sigma_{01} \\ \sigma_{01} & \sigma_{u1}^2 \end{bmatrix}$$

(Longitudinal . mixed weight Performing EM Performing gra Iteration 0: Iteration 1:	/www.stata-press analysis of pig t week    id: wee optimization adient-based opt: Log likelihood = Log likelihood =	weights) ek, covarianc imization: = -868.96185		ed)		
	Mixed-effects ML regression Group variable: id Mumber of obs = 432 Obs per groups = 48 Obs per group: min = 9 avg = 9.0					
Log likelihood	d = -868.96185			max ld chi2(1) ob > chi2	= 9 = 4649.17 = 0.0000	
weight	Coefficient St	td. err.	z P> z	[95% conf.	interval]	
week _cons					6.388399 20.13889	
Random-effe	cts parameters	Estimate	Std. err.	[95% conf.	interval]	
id: Unstructur	red var(week) var(_cons) cov(week,_cons)	.3715251 6.823363 0984378	.0812958 1.566194 .2545767	.2419532 4.351297 5973991	.570486 10.69986 .4005234	
	var(Residual) 1.596829 .123198 1.372735 1.857505					
LR test vs. 1:	inear model: chi	2(3) = 764.58		Prob > chi	2 = 0.0000	

Note: LR test is conservative and provided only for reference.

Rather than see the estimated variance components listed as variance and covariances as above, we can instead see them as correlations and standard deviations using estat sd; see [ME] estat sd.

. estat sd

Random-effects parameters	Estimate	Std. err.	[95% conf.	interval]
id: Unstructured				
sd(week)	.6095286	.0666874	.4918874	.7553052
sd(_cons)	2.612157	.2997895	2.085976	3.271064
<pre>corr(week,_cons)</pre>	0618257	.1575911	3557072	.243182
sd(Residual)	1.263657	.0487466	1.171638	1.362903

We can use estat recovariance to display the estimated variance components  $\widehat{\Sigma}$  as a correlation matrix; see [ME] estat recovariance.

```
. estat recovariance, correlation

Random-effects correlation matrix for level id

week _cons

week 1

_cons -.0618257 1
```

Finally, we can use estat wcorrelation to display the within-cluster marginal standard deviations and correlations for one of the clusters; see [ME] estat wcorrelation.

. estat wcorrelation, format(%4.2g)									
Standard deviations and correlations for id = 1:									
Standard deviations:									
obs	1	2	3	4	5	6	7	8	9
sd	2.9	3.1	3.3	3.7	4.1	4.5	5	5.5	6.1
Correlations:									
obs	1	2	3	4	5	6	7	8	9
1	1								
2	.8	1							
3	.77	.83	1						
4	.72	.81	.86	1					
5	.67	.78	.85	.89	1				
6	.63	.75	.83	.88	.91	1			
7	.59	.72	.81	.87	.91	.93	1		
8	.55	.69	.79	.86	.9	.93	.94	1	
9	.52	.66	.77	.85	.89	.92	.94	.95	1

Because within-cluster correlations can vary between clusters, estat wcorrelation by default displays the results for the first cluster. In this example, each cluster (pig) has the same number of observations, and the timings of measurements (week) are the same between clusters. Thus the within-cluster correlations are the same for all the clusters. In example 1 of [ME] estat wcorrelation, we fit a model where different clusters have different within-cluster correlations and show how to display these correlations.

We can also obtain BLUPs of the pig-level random effects  $(u_{0j} \text{ and } u_{1j})$ . We need to specify the variables to be created in the order u1 u0 because that is the order in which the corresponding variance components are listed in the output (week \_cons). We obtain the predictions and list them for the first 10 pigs.

- . predict u1 u0, reffects
- . by id, sort: generate tolist = (\_n==1)
- . list id u0 u1 if id <=10 & tolist

	id	u0	u1
1.	1	.2369444	3957636
10.	2	-1.584127	.510038
19.	3	-3.526551	.3200372
28.	4	1.964378	7719702
37.	5	1.299236	9241479
46.	6	-1.147302	5448151
55.	7	-2.590529	.0394454
64.	8	-1.137067	1696566
73.	9	-3.189545	7365507
82.	10	1.160324	.0030772

If you forget how to order your variables in predict, or if you use predict *stub*\*, remember that predict labels the generated variables for you to avoid confusion.

. describe u0	u1			
Variable name	Storage type	Display format	Value label	Variable label
u0 u1		%9.0g %9.0g		BLUP r.e. for id: _cons BLUP r.e. for id: week

Examining (1), we see that within each pig, the successive weight measurements are modeled as simple linear regression with intercept  $\beta_0 + u_{j0}$  and slope  $\beta_1 + u_{j1}$ . We can generate estimates of the pig-level intercepts and slopes with

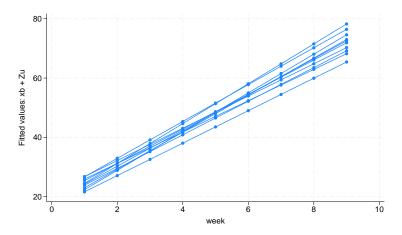
```
. generate intercept = _b[_cons] + u0
```

- . generate slope = \_b[week] + u1
- . list id intercept slope if id<=10 & tolist

	id	interc~t	slope
1.	1	19.59256	5.814132
10.	2	17.77149	6.719934
19.	3	15.82906	6.529933
28.	4	21.31999	5.437926
37.	5	20.65485	5.285748
46.	6	18.20831	5.665081
55.	7	16.76509	6.249341
64.	8	18.21855	6.040239
73.	9	16.16607	5.473345
82.	10	20.51594	6.212973

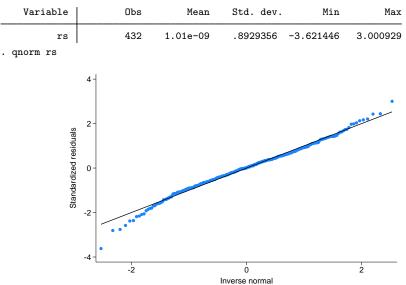
Thus we can plot estimated regression lines for each of the pigs. Equivalently, we can just plot the fitted values because they are based on both the fixed and the random effects:

- . predict fitweight, fitted
- . twoway connected fitweight week if id<=10, connect(L)



We can also generate standardized residuals and see whether they follow a standard normal distribution, as they should in any good-fitting model:

- . predict rs, rstandard
- . summarize rs



#### Example 2: Estimating the intraclass correlation

Following Rabe-Hesketh and Skrondal (2022, chap. 2), we fit a two-level random-effects model for human peak-expiratory-flow rate. The subjects were each measured twice with the Mini-Wright peak-flow meter. It is of interest to determine how reliable the meter is as a measurement device. The intraclass correlation provides a measure of reliability. Formally, in a two-level random-effects model, the intraclass correlation corresponds to the correlation of measurements within the same individual and also to the proportion of variance explained by the individual random effect.

First, we fit the two-level model with mixed:

(Peak-expirato . mixed wm    Performing EM	Www.stata-press pry-flow rate) id: optimization adient-based opt:		/pefrate, cl	ear					
Iteration 0: Log likelihood = -184.57839 Iteration 1: Log likelihood = -184.57839									
	8	= -184.57839							
Computing standard errors         Mixed-effects ML regression       Number of obs = 34         Group variable: id       Number of groups = 17         Obs per group:									
					min = 2 avg = 2.0 max = 2				
Log likelihood	d = −184.57839			Wald chi2(0) Prob > chi2					
wm	Coefficient St	td. err.	z P> z	[95% conf.	interval]				
_cons	453.9118 26	6.18617 17	.33 0.000	402.5878	505.2357				
		Г							
Random-effec	cts parameters	Estimate	Std. err.	[95% conf.	interval]				
id: Identity	var(_cons)	11458.94	3998.952	5782.176	22708.98				
	var(Residual)	396.441	135.9781	202.4039	776.4942				
LR test vs. li	inear model: chil	bar2(01) = 46	.27	Prob >= chibar	2 = 0.0000				

Now we use estat icc to estimate the intraclass correlation:

. estat icc

Intraclass correlation

Level	ICC	Std. err.	[95% conf.	interval]
id	.9665602	.0159495	.9165853	.9870185

This correlation is close to 1, indicating that the Mini-Wright peak-flow meter is reliable. But as noted by Rabe-Hesketh and Skrondal (2022), the reliability is not only a characteristic of the instrument but also of the between-subject variance. Here we see that the between-subject standard deviation,  $sd(\_cons)$ , is much larger than the within-subject standard deviation, sd(Residual).

In the presence of fixed-effects covariates, estat icc reports the residual intraclass correlation, the correlation between measurements conditional on the fixed-effects covariates. This is equivalent to the correlation of the model residuals.

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In the presence of random-effects covariates, the intraclass correlation is no longer constant and depends on the values of the random-effects covariates. In this case, estat icc reports conditional intraclass correlations assuming 0 values for all random-effects covariates. For example, in a two-level model, this conditional correlation represents the correlation of the residuals for two measurements on the same subject, which both have random-effects covariates equal to 0. Similarly to the interpretation of intercept variances in random-coefficients models (Rabe-Hesketh and Skrondal 2022, chap. 4), interpretation of this conditional intraclass correlation relies on the usefulness of the 0 baseline values of random-effects covariates. For example, mean centering of the covariates is often used to make a 0 value a useful reference.

See [ME] estat icc for more information.

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#### Example 3: Estimating residual intraclass correlations

In example 4 of [ME] **mixed**, we estimated a Cobb–Douglas production function with random intercepts at the region level and at the state-within-region level:

 $\mathbf{y}_{jk} = \mathbf{X}_{jk}\boldsymbol{\beta} + u_k^{(3)} + u_{jk}^{(2)} + \boldsymbol{\epsilon}_{jk}$ 

. use https://www.stata-press.com/data/r18/productivity (Public capital productivity)

. mixed gsp private emp hwy water other unemp || region: || state: (output omitted)

We can use estat group to see how the data are broken down by state and region:

. estat group

Group variable	No. of	Obser	vations per	group
	groups	Minimum	Average	Maximum
region	9	51	90.7	136
state	48	17	17.0	17

We are reminded that we have balanced productivity data for 17 years for each state.

We can use predict, fitted to get the fitted values

$$\widehat{\mathbf{y}}_{jk} = \mathbf{X}_{jk}\widehat{\boldsymbol{\beta}} + \widehat{u}_k^{(3)} + \widehat{u}_{jk}^{(2)}$$

but if we instead want fitted values at the region level, that is,

$$\widehat{\mathbf{y}}_{jk} = \mathbf{X}_{jk}\widehat{\boldsymbol{\beta}} + \widehat{u}_k^{(3)}$$

we need to use the relevel() option:

- . predict gsp\_region, fitted relevel(region)
- . list gsp gsp\_region in 1/10

	gsp	gsp_re~n
1.	10.25478	10.40529
2.	10.2879	10.42336
3.	10.35147	10.47343
4.	10.41721	10.52648
5.	10.42671	10.54947
6.	10.4224	10.53537
7.	10.4847	10.60781
8.	10.53111	10.64727
9.	10.59573	10.70503
10.	10.62082	10.72794

#### Technical note

Out-of-sample predictions are permitted after mixed, but if these predictions involve BLUPs of random effects, the integrity of the estimation data must be preserved. If the estimation data have changed since the mixed model was fit, predict will be unable to obtain predicted random effects that are appropriate for the fitted model and will give an error. Thus to obtain out-of-sample predictions that contain random-effects terms, be sure that the data for these predictions are in observations that augment the estimation data.

We can use estat icc to estimate residual intraclass correlations between productivity years in the same region and in the same state and region.

. estat icc

Residual intraclass correlation

Level	ICC	Std. err.	[95% conf.	interval]
region	.159893	.127627	.0287143	.5506202
state/region	.8516265	.0301733	.7823466	.9016272

estat icc reports two intraclass correlations for this three-level nested model. The first is the level-3 intraclass correlation at the region level, the correlation between productivity years in the same region. The second is the level-2 intraclass correlation at the state-within-region level, the correlation between productivity years in the same state and region.

Conditional on the fixed-effects covariates, we find that annual productivity is only slightly correlated within the same region, but it is highly correlated within the same state and region. We estimate that state and region random effects compose approximately 85% of the total residual variance.

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#### Example 4: Small-sample adjusted tests for fixed effects

To illustrate the use of test and testparm with the small option for small-sample adjusted tests for fixed effects, we refit the dental veneer data from example 14 of [ME] mixed using the Satterthwaite method (option dfmethod(satterthwaite)) to compute the DF for fixed effects.

. use https://www.stata-press.com/data/r18/veneer, clear (Dental veneer data)

. mixed gcf followup base\_gcf cda age

> || patient: followup, covariance(unstructured)

> || tooth:, reml nolog dfmethod(satterthwaite)

Mixed-effects REML regression

Number of obs = 110

Group	ing informatio	on								
Grouj	p variable		No. of groups		Obser imum	vation Ave	s per rage	group Maximum	- 1	
	patient tooth		12 55		2 2		9.2 2.0	12 2	-	
DF method: Sat	tterthwaite						DF:	mir avg maz	; = : =	10.41 28.96 50.71
Log restricted	d-likelihood :	= -4	20.92761					16.49) > F	=	1.87 0.1638
gcf	Coefficient	St	d. err.		t	P> t	[9	95% conf.	int	erval]
followup base_gcf cda age _cons	.3009815 0183127 329303 5773932 45.73862	.1 .5 .2	936863 433094 5292525 2139656 2.55497	-0. -0. -2.	13 62	0.879 0.899 0.537 0.022 0.001	3 -1 -1.	963754 3065704 39197 051598 9.90352	7.	.565717 .269945 7333636 1031885 1.57372
Random-effe	cts parameters	3	Estima	ate	Std.	err.	[9	95% conf.	int	cerval]
patient: Unstructured var(followup) var(_cons) cov(followup,_cons)			41.88 524.98 -140.42	851	253.	9997 0205 7623	20	7.38009 94.1287 70.9099	13	00.9535 350.175 .935904
tooth: Identit	ty var(_con:	5)	47.45	738	16.6	3034	2	23.8792	ę	94.3165
	var(Residua)	1)	48.86	704	10.5	0523	32	2.06479	74	1.47382
LR test vs. 1:	inear model:	chi2	2(4) = 91	.12			Pr	cob > chi2	2 =	0.0000

Note: LR test is conservative and provided only for reference.

Now we can, for example, test the hypotheses that all fixed effects are zero by typing

. testparm \*, small
( 1) [gcf]followup = 0
( 2) [gcf]base\_gcf = 0
( 3) [gcf]cda = 0
( 4) [gcf]age = 0
F( 4, 16.49) = 1.87
Prob > F = 0.1638

The F statistic for the overall test is 1.87, and the DDF is estimated to be 16.49. These results are different from the model test using the Kenward–Roger DDF method reported in the header of the estimation output in example 1 of [ME] estat df (the F statistic is 1.47, and the model DDF is 27.96).

The results differ because the Kenward–Roger method uses an adjusted F-test statistic and adjusts the fixed-effects variance–covariance estimator for a small sample. Both methods, however, lead to the same conclusion of no joint significance of the fixed effects.

Without option small, the commands test and testparm report large-sample  $\chi^2$  Wald tests. We can compare the small-sample and large-sample tests of the joint hypotheses that the coefficient on followup and the coefficient on age equal zero.

```
. test followup = age = 0, small
      [gcf]followup - [gcf]age = 0
 (1)
 (2) [gcf]followup = 0
      F(2, 10.75) =
                         3.65
           Prob > F =
                         0.0617
. test followup = age = 0
 ( 1) [gcf]followup - [gcf]age = 0
 (2)
      [gcf]followup = 0
          chi2(2) =
                         7.30
        Prob > chi2 =
                         0.0260
```

The DDF of the F test, which is computed using the Satterthwaite method from our posted results, is 10.75. The p-values are very different (0.0617 versus 0.0260), and they lead to different conclusions of whether we should reject the null hypotheses at the  $\alpha = 0.05$  level.

Similarly, you can use the small option with lincom to perform small-sample inference for linear combinations of fixed effects.

```
4
```

#### Example 5: Small-sample adjusted contrasts

As we did with test, after fitting a mixed model with the dfmethod() option for small-sample adjustment, we can use the small option with contrast to adjust for a small sample when estimating contrasts. Suppose we have collected data on a vigilance performance test. This experiment has been designed to test the response latency scores of two modes of signal during a four-hour monitoring period. This is a split-plot factorial design where signal is the whole-plot factor, hour is the subplot factor, and subject is the block factor. The whole-plot factor and the subplot factor are fixed; the block factor is random. Also, suppose that two measurements are missing in this dataset.

```
. use https://www.stata-press.com/data/r18/vptscores, clear
(Vigilance performance test scores with missing data)
```

•	tabdisp	subject	hour,	cellvar	(score)	by(signal)	concise	missing
---	---------	---------	-------	---------	---------	------------	---------	---------

Signal and Subject ID		Mon 1	itoring 2	period 3	4
Auditor	у				
	1	3	4	7	7
	2	6	5		8
	3	3	4	7	9
	4	3	3	6	8
Visual					
	5	1	2	5	10
	6	2	3	6	
	7	2	4	5	9
	8	2	3	6	11

We start by fitting a mixed model. Because the dataset is small and unbalanced, we apply the Kenward–Roger method for small-sample adjustment:

. mixed score signal##hour    subject:, reml dfmethod(kroger) nolog nogroup							
Mixed-effects REML regressionNumber of obs = 30DF method: Kenward-RogerDF:min = 16.02avg = 16.76max = 18.29F(7, 16.08) = 43.84Frob > F = 0.0000							
score	Coefficient S	td. err.	t	P> t	[95% conf.	interval]	
signal Visual	-2 .0	6288677	-3.18	0.005	-3.319693	6803071	
hour 2 3 4	3.108222 .	5359916 5911044 5359916	0.47 5.26 7.93	0.647 0.000 0.000	8861371 1.859163 3.113863	1.386137 4.357281 5.386137	
signal#hour Visual#2 Visual#3 Visual#4 _cons	.6417778 . 4.044205 .	7580066 7979294 7979294 4446766	1.32 0.80 5.07 8.43	0.206 0.433 0.000 0.000	6067405 -1.046666 2.355762 2.816836	2.606741 2.330221 5.732649 4.683164	
Random-effec	cts parameters	Estima	te Sto	l. err.	[95% conf.	interval]	
subject: Identity var(_cons) .2163751 .2345718 .0258477 1.811312						1.811312	
	var(Residual) .574574 .2062107 .2843515 1.161011						
LR test vs. linear model: chibar2(01) = 1.55 Prob >= chibar2 = 0.1069							

We can test the main effects and the interaction effects by typing the contrast command. With the small option, contrast reports small-sample adjusted F tests. Without the small option, contrast performs large-sample  $\chi^2$  Wald tests. Below is the comparison of the small-sample and the large-sample contrasts:

Contrasts of marginal linear predictions								
Margins: asbalanced								
		df	ddf	F	P>F			
score								
signal		1	5.95	1.78	0.2307			
hour		3	16.35	100.62	0.0000			
signal#hour		3	16.35	9.66	0.0007			
. contrast sig	gnal##hou	ır						
Contrasts of m	narginal	linear	predictions					
Margins: asbal	lanced							
		df	chi2	P>chi2				
score signal		1	1.79	0.1810				

. contrast signal##hour, small

	df	chi2	P>chi2
score signal	1	1.79	0.1810
hour	3	304.95	0.0000
signal#hour	3	29.35	0.0000

From these results, we can see that the p-values for the main effect of signal and the interaction effect vary between small-sample and large-sample tests. However, both tests indicate that the hour effect and the interaction effects are significant. We can decompose the interaction effect into separate interaction contrasts for further investigation.

contrast r.signal#ar.hour, small
 Contrasts of marginal linear predictions
 Margins: asbalanced

vs Auditory)

(4 vs 3)

			df	ddf	F	P>F
score						
	signal#h	nour				
(Visual vs Aud	ditory) (2 vs	1)	1	16.02	1.74	0.2056
(Visual vs Aud	ditory) (3 vs	2)	1	16.37	0.6594	
(Visual vs Aud	ditory) (4 vs	3)	1	16.66	16.57	0.0008
	Joi	int	3	16.35	9.66	0.0007
	Contrast	Std. err.		df	[95% conf.	interval]
score						
signal#hour						
(Visual						
vs						
Auditory)						
(2 vs 1)	1	.7580066		16.0	6067405	2.606741
(Visual						
vs						
Auditory)						
(3 vs 2)	3582222	.7979294		16.4	-2.046666	1.330221
(Visual						

From previous analysis, we already knew the overall interaction was significant. From the decomposition, we can easily see that the overall significance is driven by differences in the third and fourth hours; the change in response latency from hour three to hour four is greater for visual signals than for auditory signals.

.8359478

16.7

1.635991

5.168863

We can also calculate the pairwise differences of the hourly marginal means by typing the pwcompare command. With the small option, pwcompare reports small-sample adjusted pairwise comparisons along with the degrees of freedom for each pairwise comparison.

```
. pwcompare hour, small
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

3.402427

	Contrast	Std. err.	df	5	usted interval]
score					
hour					
2 vs 1	.75	.3790033	16.0	0533703	1.55337
3 vs 1	3.429111	.3989647	16.4	2.584889	4.273333
4 vs 1	6.272103	.3989647	16.4	5.427881	7.116324
3 vs 2	2.679111	.3989647	16.4	1.834889	3.523333
4 vs 2	5.522103	.3989647	16.4	4.677881	6.366324
4 vs 3	2.842991	.4179739	16.7	1.959774	3.726209

When we compare these results with the large-sample results below, we can see that the confidence interval of hour 2 versus hour 1 changes to include 0. Therefore, after adjusting for small-sample

size, we would not reject the hypothesis that the means for hour 1 and hour 2 are equivalent at the 5% significance level.

```
. pwcompare hour
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	Unadjusted [95% conf. interval]
score			
hour			
2 vs 1	.75	.3790033	.0071672 1.492833
3 vs 1	3.429111	.3971529	2.650706 4.207516
4 vs 1	6.272103	.3971529	5.493697 7.050508
3 vs 2	2.679111	.3971529	1.900706 3.457516
4 vs 2	5.522103	.3971529	4.743697 6.300508
4 vs 3	2.842991	.4145085	2.03057 3.655413

## Stored results

pwcompare with option small stores the following in r():

Ma	trices	
1410	uices	

r(L_df)	degrees	of	freedom	for	each	margin	difference
r(M_df)	degrees	of	freedom	for	each	margin	estimate

pwcompare with options post and small stores the following in e():

Matrices

e(L_df)	degrees	of	freedom	for	each	margin	difference
e(M_df)	degrees	of	freedom	for	each	margin	estimate

## Methods and formulas

Methods and formulas are presented under the following headings:

Prediction Small-sample inference

### Prediction

Following the notation defined throughout [ME] mixed, BLUPs of random effects u are obtained as

$$\widetilde{\mathbf{u}} = \widetilde{\mathbf{G}} \mathbf{Z}' \widetilde{\mathbf{V}}^{-1} \left( \mathbf{y} - \mathbf{X} \widehat{\boldsymbol{\beta}} \right)$$

where  $\widetilde{\mathbf{G}}$  and  $\widetilde{\mathbf{V}}$  are  $\mathbf{G}$  and  $\mathbf{V} = \mathbf{Z}\mathbf{G}\mathbf{Z}' + \sigma_{\epsilon}^{2}\mathbf{R}$  with maximum likelihood (ML) or REML estimates of the variance components plugged in. Standard errors for BLUPs are calculated based on the iterative technique of Bates and Pinheiro (1998, sec. 3.3) for estimating the BLUPs themselves. If estimation is done by REML, these standard errors account for uncertainty in the estimate of  $\beta$ , while for ML the standard errors treat  $\beta$  as known. As such, standard errors of REML-based BLUPs will usually be larger.

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Fitted values are given by  $\mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\tilde{\mathbf{u}}$ , residuals as  $\hat{\boldsymbol{\epsilon}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}} - \mathbf{Z}\tilde{\mathbf{u}}$ , and standardized residuals as

$$\widehat{\boldsymbol{\epsilon}}_* = \widehat{\sigma}_{\boldsymbol{\epsilon}}^{-1} \widehat{\mathbf{R}}^{-1/2} \widehat{\boldsymbol{\epsilon}}$$

If the relevel(*levelvar*) option is specified, fitted values, residuals, and standardized residuals consider only those random-effects terms up to and including level *levelvar* in the model.

For details concerning the calculation of scores, see Methods and formulas in [ME] mixed.

#### Small-sample inference

For small-sample computations performed when the small option is used with test, testparm, lincom, contrast, or pwcompare, see *Denominator degrees of freedom* in *Methods and formulas* of [ME] **mixed**.

### References

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- Rabe-Hesketh, S., and A. Skrondal. 2022. *Multilevel and Longitudinal Modeling Using Stata.* 4th ed. College Station, TX: Stata Press.

## Also see

[ME] **mixed** — Multilevel mixed-effects linear regression

[U] 20 Estimation and postestimation commands

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