Glossary

ANOVA denominator degrees of freedom (DDF) method. This method uses the traditional ANOVA for computing DDF. According to this method, the DDF for a test of a fixed effect of a given variable depends on whether that variable is also included in any of the random-effects equations. For traditional ANOVA models with balanced designs, this method provides exact sampling distributions of the test statistics. For more complex mixed-effects models or with unbalanced data, this method typically leads to poor approximations of the actual sampling distributions of the test statistics.

approximation denominator degrees of freedom (DDF) methods. The Kenward–Roger and Satterthwaite DDF methods are referred to as approximation methods because they approximate the sampling distributions of test statistics using t and F distributions with the DDF specific to the method for complicated mixed-effects models and for simple mixed models with unbalanced data. Also see *exact denominator degrees of freedom (DDF) methods*.

between–within denominator degrees of freedom (DDF) method. See *repeated denominator degrees of freedom (DDF) method*.

BLUPs. BLUPs are best linear unbiased predictions of either random effects or linear combinations of random effects. In linear models containing random effects, these effects are not estimated directly but instead are integrated out of the estimation. Once the fixed effects and variance components have been estimated, you can use these estimates to predict group-specific random effects. These predictions are called BLUPs because they are unbiased and have minimal mean squared errors among all linear functions of the response.

canonical link. Corresponding to each family of distributions in a generalized linear model (GLM) is a canonical link function for which there is a sufficient statistic with the same dimension as the number of parameters in the linear predictor. The use of canonical link functions provides the GLM with desirable statistical properties, especially when the sample size is small.

conditional hazard function. In the context of mixed-effects survival models, the conditional hazard function is the hazard function computed conditionally on the random effects. Even within the same covariate pattern, the conditional hazard function varies among individuals who belong to different random-effects clusters.

conditional hazard ratio. In the context of mixed-effects survival models, the conditional hazard ratio is the ratio of two conditional hazard functions evaluated at different values of the covariates. Unless stated differently, the denominator corresponds to the conditional hazard function at baseline, that is, with all the covariates set to zero.

conditional overdispersion. In a negative binomial mixed-effects model, conditional overdispersion is overdispersion conditional on random effects. Also see *overdispersion*.

containment denominator degrees of freedom (DDF) method. See *ANOVA denominator degrees of freedom (DDF) method*.

continuous-time autoregressive structure. A generalization of the autoregressive structure that allows for unequally spaced and noninteger time values.

covariance structure. In a mixed-effects model, covariance structure refers to the variance–covariance structure of the random effects.

crossed-effects model. A crossed-effects model is a mixed-effects model in which the levels of random effects are not nested. A simple crossed-effects model for cross-sectional time-series data would contain a random effect to control for panel-specific variation and a second random effect to control for time-specific random variation. Rather than being nested within panel, in this model a random effect due to a given time is the same for all panels.

crossed-random effects. See *crossed-effects model*.

EB. See *empirical Bayes*.

empirical Bayes. In generalized linear mixed-effects models, empirical Bayes refers to the method of prediction of the random effects after the model parameters have been estimated. The empirical Bayes method uses Bayesian principles to obtain the posterior distribution of the random effects, but instead of assuming a prior distribution for the model parameters, the parameters are treated as given.

empirical Bayes mean. See *posterior mean*.

empirical Bayes mode. See *posterior mode*.

error covariance, error covariance structure. Variance–covariance structure of the errors within the lowest-level group. For example, if you are modeling random effects for classes nested within schools, then error covariance refers to the variance–covariance structure of the observations within classes, the lowest-level groups. With a slight abuse of the terminology, error covariance is sometimes also referred to as residual covariance or residual error covariance in the literature.

exact denominator degrees of freedom (DDF) methods. Residual, repeated, and ANOVA DDF methods are referred to as exact methods because they provide exact t and F sampling distributions of test statistics for special classes of mixed-effects models—linear regression, repeated-measures designs, and traditional ANOVA models—with balanced data. Also see *approximation denominator degrees of freedom (DDF) methods*.

fixed effects. In the context of multilevel mixed-effects models, fixed effects represent effects that are constant for all groups at any level of nesting. In the ANOVA literature, fixed effects represent the levels of a factor for which the inference is restricted to only the specific levels observed in the study. See also *fixed-effects model* in [XT] **Glossary**.

free parameter. Free parameters are parameters that are not defined by a linear form. Free parameters are displayed with a forward slash in front of their names or their equation names.

Gauss–Hermite quadrature. In the context of generalized linear mixed models, Gauss–Hermite quadrature is a method of approximating the integral used in the calculation of the log likelihood. The quadrature locations and weights for individual clusters are fixed during the optimization process.

generalized linear mixed-effects model. A generalized linear mixed-effects model is an extension of a generalized linear model allowing for the inclusion of random deviations (effects).

generalized linear model. The generalized linear model is an estimation framework in which the user specifies a distributional family for the dependent variable and a link function that relates the dependent variable to a linear combination of the regressors. The distribution must be a member of the exponential family of distributions. The generalized linear model encompasses many common models, including linear, probit, and Poisson regression.

GHQ. See *Gauss–Hermite quadrature*.

GLM. See *generalized linear model*.

GLME model. See *generalized linear mixed-effects model*.

GLMM. Generalized linear mixed model. See *generalized linear mixed-effects model*.

hierarchical model. A hierarchical model is one in which successively more narrowly defined groups are nested within larger groups. For example, in a hierarchical model, patients may be nested within doctors who are in turn nested within the hospital at which they practice.

intraclass correlation. In the context of mixed-effects models, intraclass correlation refers to the correlation for pairs of responses at each nested level of the model.

Kenward–Roger denominator degrees of freedom (DDF) method. This method implements the Kenward and Roger (1997) method, which is designed to approximate unknown sampling distributions of test statistics for complex linear mixed-effects models. This method is supported only with restricted maximum-likelihood estimation.

Laplacian approximation. Laplacian approximation is a technique used to approximate definite integrals without resorting to quadrature methods. In the context of mixed-effects models, Laplacian approximation is as a rule faster than quadrature methods at the cost of producing biased parameter estimates of variance components.

Lindstrom–Bates algorithm. An algorithm used by the linearization method.

linear form. A linear combination is what we call a "linear form" as long as you do not refer to its coefficients or any subset of the linear combination anywhere in the expression. Linear forms are beneficial for some nonlinear commands such as `nl` because they make derivative computation faster and more accurate. In contrast to free parameters, parameters of a linear form are displayed without forward slashes in the output. Rather, they are displayed as parameters within an equation whose name is the linear combination name. Also see *Linear forms versus linear combinations* in [ME] **menl**.

linear mixed model. See *linear mixed-effects model*.

linear mixed-effects model. A linear mixed-effects model is an extension of a linear model allowing for the inclusion of random deviations (effects).

linearization log likelihood. Objective function used by the linearization method for optimization. This is the log likelihood of the linear mixed-effects model used to approximate the specified nonlinear mixed-effects model.

linearization method, **Lindstrom–Bates method**. Method developed by Lindstrom and Bates (1990) to approximate for fitting nonlinear mixed-effects models. The linearization method uses a first-order Taylor-series expansion of the specified nonlinear mean function to approximate it with a linear function of fixed and random effects. Thus a nonlinear mixed-effects model is approximated by a linear mixed-effects model, in which the fixed-effects and random-effects design matrices involve derivatives of the nonlinear mean function with respect to fixed effects (coefficients) and random effects, respectively. Also see *Introduction* in [ME] **menl**.

link function. In a generalized linear model or a generalized linear mixed-effects model, the link function relates a linear combination of predictors to the expected value of the dependent variable. In a linear regression model, the link function is simply the identity function.

LME model. See *linear mixed-effects model*.

lowest-level group. The second level of a multilevel model with the observations composing the first level. For example, if you are modeling random effects for classes nested within schools, then classes are the lowest-level groups.

MCAGH. See *mode-curvature adaptive Gauss–Hermite quadrature*.

mean–variance adaptive Gauss–Hermite quadrature. In the context of generalized linear mixed models, mean–variance adaptive Gauss–Hermite quadrature is a method of approximating the integral used in the calculation of the log likelihood. The quadrature locations and weights for individual clusters are updated during the optimization process by using the posterior mean and the posterior standard deviation.

mixed model. See *mixed-effects model*.

mixed-effects model. A mixed-effects model contains both fixed and random effects. The fixed effects are estimated directly, whereas the random effects are summarized according to their (co)variances. Mixed-effects models are used primarily to perform estimation and inference on the regression coefficients in the presence of complicated within-subject correlation structures induced by multiple levels of grouping.

mode-curvature adaptive Gauss–Hermite quadrature. In the context of generalized linear mixed models, mode-curvature adaptive Gauss–Hermite quadrature is a method of approximating the integral used in the calculation of the log likelihood. The quadrature locations and weights for individual clusters are updated during the optimization process by using the posterior mode and the standard deviation of the normal density that approximates the log posterior at the mode.

MVAGH. See *mean–variance adaptive Gauss–Hermite quadrature*.

named substitutable expression. A named substitutable expression is a substitutable expression defined within `menl`'s `define()` option; see *Substitutable expressions* in [ME] **menl**.

nested random effects. In the context of mixed-effects models, nested random effects refer to the nested grouping factors for the random effects. For example, we may have data on students who are nested in classes that are nested in schools.

NLME model. See *nonlinear mixed-effects model*.

nonlinear mixed-effects model. A model in which the conditional mean function given random effects is a nonlinear function of fixed and random effects. A linear mixed-effects model is a special case of a nonlinear mixed-effects model.

one-level model. A one-level model has no multilevel structure and no random effects. Linear regression is a one-level model.

overdispersion. In count-data models, overdispersion occurs when there is more variation in the data than would be expected if the process were Poisson.

posterior mean. In generalized linear mixed-effects models, posterior mean refers to the predictions of random effects based on the mean of the posterior distribution.

posterior mode. In generalized linear mixed-effects models, posterior mode refers to the predictions of random effects based on the mode of the posterior distribution.

QR decomposition. QR decomposition is an orthogonal-triangular decomposition of an augmented data matrix that speeds up the calculation of the log likelihood; see *Methods and formulas* in [ME] **mixed** for more details.

quadrature. Quadrature is a set of numerical methods to evaluate a definite integral.

random coefficient. In the context of mixed-effects models, a random coefficient is a counterpart to a slope in the fixed-effects equation. You can think of a random coefficient as a randomly varying slope at a specific level of nesting.

random effects. In the context of mixed-effects models, random effects represent effects that may vary from group to group at any level of nesting. In the ANOVA literature, random effects represent the levels of a factor for which the inference can be generalized to the underlying population represented by the levels observed in the study. See also *random-effects model* in [XT] **Glossary**.

random intercept. In the context of mixed-effects models, a random intercept is a counterpart to the intercept in the fixed-effects equation. You can think of a random intercept as a randomly varying intercept at a specific level of nesting.

random-effects substitutable expression. A random-effects substitutable expression is a substitutable expression containing random-effects terms; see *Random-effects substitutable expressions* in [ME] **menl**.

REML. See *restricted maximum likelihood*.

repeated denominator degrees of freedom (DDF) method. This method uses the repeated-measures ANOVA for computing DDF. It is used with balanced repeated-measures designs with spherical correlation error structures. It partitions the residual degrees of freedom into the between-subject degrees of freedom and the within-subject degrees of freedom. The repeated method is supported only with two-level models. For more complex mixed-effects models or with unbalanced data, this method typically leads to poor approximations of the actual sampling distributions of the test statistics.

residual covariance, **residual error covariance**. See *error covariance*.

residual denominator degrees of freedom (DDF) method. This method uses the residual degrees of freedom, $n - \text{rank}(X)$, as the DDF for all tests of fixed effects. For a linear model without random effects with independent and identically distributed errors, the distributions of the test statistics for fixed effects are t or F distributions with the residual DDF. For other mixed-effects models, this method typically leads to poor approximations of the actual sampling distributions of the test statistics.

restricted maximum likelihood. Restricted maximum likelihood is a method of fitting linear mixed-effects models that involves transforming out the fixed effects to focus solely on variance–component estimation.

Satterthwaite denominator degrees of freedom (DDF) method. This method implements a generalization of the Satterthwaite (1946) approximation of the unknown sampling distributions of test statistics for complex linear mixed-effects models. This method is supported only with restricted maximum-likelihood estimation.

substitutable expression. Substitutable expressions are like any other mathematical expressions involving scalars and variables, such as those you would use with Stata's `generate` command, except that the parameters to be estimated are bound in braces. See *Substitutable expressions* in [ME] **menl**.

three-level model. A three-level mixed-effects model has one level of observations and two levels of grouping. Suppose that you have a dataset consisting of patients overseen by doctors at hospitals, and each doctor practices at one hospital. Then a three-level model would contain a set of random effects to control for hospital-specific variation, a second set of random effects to control for doctor-specific random variation within a hospital, and a random-error term to control for patients' random variation.

two-level model. A two-level mixed-effects model has one level of observations and one level of grouping. Suppose that you have a panel dataset consisting of patients at hospitals; a two-level model would contain a set of random effects at the hospital level (the second level) to control for hospital-specific random variation and a random-error term at the observation level (the first level) to control for within-hospital variation.

variance components. In a mixed-effects model, the variance components refer to the variances and covariances of the various random effects.

within-group errors. In a two-level model with observations nested within groups, within-group errors refer to error terms at the observation level. In a higher-level model, they refer to errors within the lowest-level groups.

References

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