## Title

qrinv( ) - Generalized inverse of matrix via QR decomposition

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## Description

qrinv $(A, \ldots)$ returns the inverse or generalized inverse of real or complex matrix $A: m \times n, m \geq$ $n$. If optional argument rank is specified, the rank of $A$ is returned there.
_qrinv $(A, \ldots)$ does the same thing except that, rather than returning the result, it overwrites the original matrix $A$ with the result. _qrinv() returns the rank of $A$.

In both cases, optional argument tol specifies the tolerance for determining singularity; see Remarks and examples below.

## Syntax

```
numeric matrix qrinv(numeric matrix A)
numeric matrix qrinv(numeric matrix A, rank)
numeric matrix qrinv(numeric matrix A, rank, real scalar tol)
real scalar _qrinv(numeric matrix A)
real scalar _qrinv(numeric matrix A, real scalar tol)
```

where the type of rank is irrelevant; the rank of $A$ is returned there.

## Remarks and examples

qrinv() and _qrinv() are most often used on square and possibly rank-deficient matrices but may be used on nonsquare matrices that have more rows than columns. Also see [M-5] pinv() for an alternative. See [M-5] luinv() for a more efficient way to obtain the inverse of full-rank, square matrices, and see [M-5] invsym() for inversion of real, symmetric matrices.

When $A$ is of full rank, the inverse calculated by qrinv() is essentially the same as that computed by the faster luinv(). When $A$ is singular, qrinv() and _qrinv() compute a generalized inverse, $A^{*}$, which satisfies

$$
\begin{aligned}
A\left(A^{*}\right) A & =A \\
\left(A^{*}\right) A\left(A^{*}\right) & =A^{*}
\end{aligned}
$$

This generalized inverse is also calculated for nonsquare matrices that have more rows than columns and, then returned is a least-squares solution. If $A$ is $m \times n, m \geq n$, and if the rank of $A$ is equal to $n$, then $\left(A^{*}\right) A=I$, ignoring roundoff error.
qrinv ( $A$ ) is implemented as qrsolve (A, I (rows (A))); see [M-5] qrsolve() for details and for use of the optional tol argument.

## Conformability

qrinv( $A$, rank, tol):
input:
A: $\quad m \times n, \quad m \geq n$
tol: $\quad 1 \times 1 \quad$ (optional)
output:
rank: $1 \times 1 \quad$ (optional)
result: $\quad n \times m$
_qrinv ( $A$, tol $)$ :
input:

$$
A: \quad m \times n, \quad m \geq n
$$

tol: $1 \times 1 \quad$ (optional)
output:
A: $\quad n \times m$
result: $1 \times 1 \quad$ (containing rank)

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex.
qrinv $(A, \ldots)$ and _qrinv $(A, \ldots)$ return a result containing missing values if $A$ contains missing values.
-qrinv $(A, \ldots)$ aborts with error if $A$ is a view.
See [M-5] qrsolve( ) and [M-1] Tolerance for information on the optional tol argument.

## Also see

[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion
[M-5] invsym( ) - Symmetric real matrix inversion
[M-5] luinv() - Square matrix inversion
[M-5] pinv() - Moore-Penrose pseudoinverse
[M-5] qrsolve( ) - Solve AX=B for X using QR decomposition
[M-5] solve_tol( ) - Tolerance used by solvers and inverters
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse
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