## Title

IdI() — Bunch-Kaufman decomposition

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## Description

$\operatorname{ldl}(A, L, D, p)$ returns the Bunch-Kaufman decomposition (with diagonal pivoting) of $A$ in a permuted lower-triangular matrix $L$ and a symmetric block-diagonal matrix $D$ with $1 \times 1$ and $2 \times 2$ diagonal blocks, along with a permutation vector $p$.

With the permutation vector, $L[p,$.$] becomes a lower-triangular matrix with unit diagonal.$
Up to roundoff error, the returned results are such that

$$
A[p, p]=L[p, .] * D * L[p, .]^{\prime}
$$

$\operatorname{ldl}(A, L, D)$ is similar to $\operatorname{ldl}(A, L, D, p)$, but the permutation vector $p$ is omitted from the output.

Up to roundoff error, the returned results are such that

$$
A=L * D * L^{\prime}
$$

## Syntax

void ldl (numeric matrix $A, L, D)$
void 1 dl (numeric matrix $A, L, D, p$ )
where

1. $A$ is symmetric (Hermitian) indefinite.
2. the types of $L, D$, and $p$ are irrelevant; results are returned there.

## Remarks and examples

The Bunch-Kaufman decomposition is a generalization of the Cholesky decomposition.
Bunch-Kaufman decomposition of matrix $A$ can be written as

$$
P A P^{\prime}=P L D L^{\prime} P^{\prime}
$$

where $P$ is a permutation matrix that permutes the rows of $A$.
$L$ is the permuted lower-triangular matrix. With the permutation matrix $P, P L$ is a lower-triangular matrix with unit diagonal.
$D$ is the symmetric block-diagonal matrix $D$ with $1 \times 1$ and $2 \times 2$ diagonal blocks.
Rather than returning $P$ directly, returned is $p$ corresponding to $P$. Lowercase $p$ is a column vector that contains the subscripts of the rows in the desired order. That is,

$$
P L=L[p, .]
$$

The advantage of this is that $p$ requires less memory than $P$, and the reorganization, should it be desired, can be performed more quickly; see [M-1] Permutation.

## > Example 1: Bunch-Kaufman decomposition

The Bunch-Kaufman decomposition of $A$ can be written as

$$
A=L * D * L^{\prime}
$$

$\operatorname{ldl}(A, L, D)$ will make this calculation:

```
: A
[symmetric]
    1 2 3
    l|lll
: ldl(A, L = ., D = ., p = .)
: L
```

            \(1 \quad 2\)
                                    23
    \begin{tabular}{|rrr|}
    \hline 1 \& 0 \& 0 <br>
-.1666666667 \& .8333333333 \& 1 <br>
0 \& 1 \& 0 <br>
\hline
\end{tabular}

: D
[symmetric]
$1 \quad 2$
3

|  | 2 |  |  |
| :--- | :--- | :--- | :--- |
| 2 |  |  |  |
| 3 | 4 | 2 |  |
|  | 0 | 0 | 1.666666667 |

: p
1
1
$2 \quad 3$
32
: mreldif(A, L * D * L')
$1.11022 \mathrm{e}-16$
: mreldif(A[p, p], L[p, .] * D * L[p, .]')
$1.11022 \mathrm{e}-16$

## Conformability

$\operatorname{ldl}(A, L, D, p)$ :
input:
A: $\quad n \times n$
output:
L: $\quad n \times n$
D: $\quad n \times n$
p: $n \times 1 \quad$ (optional)

## Diagnostics

$\operatorname{ldl}(A, L, D, p)$ returns missing results if $A$ contains missing values.

## Also see

[M-5] cholesky () - Cholesky square-root decomposition
[M-4] Matrix - Matrix functions

[^0]For suggested citations, see the FAQ on citing Stata documentation.


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