

**gschurd()** — Generalized Schur decomposition

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## Description

`gschurd(A, B, T, R, U, V, w, b)` computes the generalized Schur decomposition of two square, numeric matrices,  $A$  and  $B$ , and the [generalized eigenvalues](#). The decomposition is returned in the [Schur-form](#) matrix,  $T$ ; the upper-triangular matrix,  $R$ ; and the orthogonal (unitary) matrices,  $U$  and  $V$ . The generalized eigenvalues are returned in the complex vectors  $w$  and  $b$ .

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m)` computes the generalized Schur decomposition of two square, numeric matrices,  $A$  and  $B$ , and the [generalized eigenvalues](#), and groups the results according to whether a condition on each generalized eigenvalue is satisfied.  $f$  is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed [below](#). The number of generalized eigenvalues for which the condition is true is returned in  $m$ .

`_gschurd()` mirrors `gschurd()`, the difference being that it returns  $T$  in  $A$  and  $R$  in  $B$ .

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns  $T$  in  $A$  and  $R$  in  $B$ .

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [\[M-1\] LAPACK](#). Their direct use is not recommended.

## Syntax

```
void          gschurd(A, B, T, R, U, V, w, b)
```

```
void          _gschurd(A, B, U, V, w, b)
```

```
void gschurdgroupby(A, B, f, T, R, U, V, w, b, m)
```

```
void _gschurdgroupby(A, B, f, U, V, w, b, m)
```

## Remarks and examples

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Remarks are presented under the following headings:

*Generalized Schur decomposition*  
*Grouping the results*

### Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices,  $A$  and  $B$ , can be written as

$$U' \times A \times V = T$$

$$U' \times B \times V = R$$

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where **T** is in Schur form, **R** is upper triangular, and **U** and **V** are orthogonal if **A** and **B** are real and are unitary if **A** or **B** is complex. The complex vectors **w** and **b** contain the generalized eigenvalues.

If **A** and **B** are real, **T** is in real Schur form and **R** is a real upper-triangular matrix. If **A** or **B** is complex, **T** is in complex Schur form and **R** is a complex upper-triangular matrix.

In the example below, we define **A** and **B**, obtain the generalized Schur decomposition, and list **T** and **R**.

```
: A = (6, 2, 8, -1\ -3, -4, -6, 4\ 0, 8, 4, 1\ -8, -7, -3, 5)
: B = (8, 0, -8, -1\ -6, -2, -6, -1\ -7, -6, 2, -6\ 1, -7, 9, 2)
: gschurd(A, B, T=., R=., U=., V=., w=., b=.)
```

```
: T
      1          2          3          4
1  12.99313938  1.746927947  3.931212285 -10.91622337
2           0    .014016016  6.153566902  1.908835695
3           0  -4.362999645  1.849905717 -2.998194791
4           0           0           0 -5.527285433
```

```
: R
      1          2          3          4
1  4.406836593  6.869534063 -1.840892081  1.740906311
2           0   13.88730687  0 -6.6995556735
3           0           0  9.409495218 -4.659386723
4           0           0           0  9.453808732
```

```
: w
      1          2          3          4
1  12.9931394  .409611804+1.83488354i  .024799819-.111092453i -5.52728543
```

```
: b
      1          2          3          4
1  4.406836593  4.145676341  .2509986829  9.453808732
```

Generalized eigenvalues can be obtained by typing

```
: w:/b
      1          2          3          4
1  2.94840508  .098804579+.442601735i  .098804579-.442601735i -5.84662287
```

### Grouping the results

`gschurdgroupby()` reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair **w** and **b**. It also reorders the generalized Schur form **T**, **R**, and orthogonal (unitary) matrices, **U** and **V**, correspondingly.

We must pass `gschurdgroupby()` a [pointer](#) to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `gschurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```

: real scalar onlyreal(complex scalar w, real scalar b)
> {
>     if(b==0) return(0)
>     if(Im(w/b)==0) return(1)
>     return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)

```

We obtain

```

: T
      1      2      3      4
1  12.99313938  8.19798168  6.285710813  5.563547054
2      0 -5.952366071 -1.473533834  2.750066482
3      0      0 -0.2015830885  3.882051743
4      0      0  6.337230739  1.752690714

: R
      1      2      3      4
1  4.406836593  2.267479575 -6.745927817  1.720793701
2      0  10.18086202 -2.253089622  5.74882307
3      0      0 -12.5704981  0
4      0      0      0  9.652818299

: w
      1      2      3      4
1  12.9931394 -5.95236607 .36499234+1.63500766i .36499234-1.63500766i

: b
      1      2      3      4
1  4.406836593  10.18086202  3.694083258  3.694083258

: w:/b
      1      2      3      4
1  2.94840508 -.584662287 .098804579+.442601735i .098804579-.442601735i

```

`m` contains the number of real, generalized eigenvalues

```

: m
2

```

## Conformability

`gschurd(A, B, T, R, U, V, w, b):`

*input:*

*A:*     $n \times n$   
*B:*     $n \times n$

*output:*

*T:*     $n \times n$   
*R:*     $n \times n$   
*U:*     $n \times n$   
*V:*     $n \times n$   
*w:*     $1 \times n$   
*b:*     $1 \times n$

`_gschurd(A, B, U, V, w, b):`

*input:*

*A:*     $n \times n$   
*B:*     $n \times n$

*output:*

*A:*     $n \times n$   
*B:*     $n \times n$   
*U:*     $n \times n$   
*V:*     $n \times n$   
*w:*     $1 \times n$   
*b:*     $1 \times n$

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m):`

*input:*

*A:*     $n \times n$   
*B:*     $n \times n$   
*f:*     $1 \times 1$

*output:*

*T:*     $n \times n$   
*R:*     $n \times n$   
*U:*     $n \times n$   
*V:*     $n \times n$   
*w:*     $1 \times n$   
*b:*     $1 \times n$   
*m:*     $1 \times 1$

`_gschurdgroupby(A, B, f, U, V, w, b, m)`:

*input:*

*A:*  $n \times n$

*B:*  $n \times n$

*f:*  $1 \times 1$

*output:*

*A:*  $n \times n$

*B:*  $n \times n$

*U:*  $n \times n$

*V:*  $n \times n$

*w:*  $1 \times n$

*b:*  $1 \times n$

*m:*  $1 \times 1$

## Diagnostics

`_gschurd()` and `_gschurdgroupby()` abort with error if *A* or *B* is a view.

`gschurd()`, `_gschurd()`, `gschurdgroupby()`, and `_gschurdgroupby()` return missing results if *A* or *B* contains missing values.

## Also see

[M-1] **LAPACK** — Linear algebra package (LAPACK) routines

[M-5] **geigensystem()** — Generalized eigenvectors and eigenvalues

[M-5] **ghessenbergd()** — Generalized Hessenberg decomposition

[M-4] **Matrix** — Matrix functions

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