cholsolve( ) — Solve AX=B for X using Cholesky decomposition

Description

`cholsolve(`A`,` B`)` solves $AX = B$ and returns X for symmetric (Hermitian), positive-definite A. `cholsolve()` returns a matrix of missing values if A is not positive definite or if A is singular.

`cholsolve(`A`,` B`,` *tol*`)` does the same thing; it allows you to specify the tolerance for declaring that A is singular; see *Tolerance* under *Remarks and examples* below.

`_cholsolve(`A`,` B`)` and `_cholsolve(`A`,` B`,` *tol*`)` do the same thing, except that, rather than returning the solution X, they overwrite B with the solution, and in the process of making the calculation, they destroy the contents of A.

`cholsolvelapacke(`A`)`, `cholsolvelapacke(`A`,` *tol*`)`, `_cholsolvelapacke(`A`)`, and `_cholsolvelapacke(`A`,` *tol*`)` are similar to their correspondent functions without `lapacke` endings, but instead they use interfaces to the LAPACK routines to compute the solutions.

Syntax

numeric matrix `cholsolve(`*numeric matrix A*`,` *numeric matrix B*`)`

numeric matrix `cholsolve(`*numeric matrix A*`,` *numeric matrix B*`,` *real scalar tol*`)`

void `_cholsolve(`*numeric matrix A*`,` *numeric matrix B*`)`

void `_cholsolve(`*numeric matrix A*`,` *numeric matrix B*`,` *real scalar tol*`)`

numeric matrix `cholsolvelapacke(`*numeric matrix A*`,` *numeric matrix B*`)`

numeric matrix `cholsolvelapacke(`*numeric matrix A*`,` *numeric matrix B*`,`
real scalar tol`)`

void `_cholsolvelapacke(`*numeric matrix A*`,` *numeric matrix B*`)`

void `_cholsolvelapacke(`*numeric matrix A*`,` *numeric matrix B*`,`
real scalar tol`)`

Remarks and examples

The above functions solve $AX = B$ via Cholesky decomposition and are accurate. When A is not symmetric and positive definite, [M-5] **lusolve( )**, [M-5] **qrsolve( )**, and [M-5] **svsolve( )** are alternatives based on the LU decomposition, the QR decomposition, and the singular value decomposition (SVD). The alternatives differ in how they handle singular A. Then, the LU-based routines return missing values, whereas the QR-based and SVD-based routines return generalized (least-squares) solutions.

Remarks are presented under the following headings:

Derivation

We wish to solve for X

$$AX = B \tag{1}$$

when A is symmetric and positive definite. Perform the Cholesky decomposition of A so that we have $A = GG'$. Then, (1) can be written as

$$GG'X = B \tag{2}$$

Define

$$Z = G'X \tag{3}$$

Then, (2) can be rewritten as

$$GZ = B \tag{4}$$

It is easy to solve (4) for Z because G is a lower-triangular matrix. Once Z is known, it is easy to solve (3) for X because G' is upper triangular.

Relationship to inversion

See *Relationship to inversion* in [M-5] **lusolve( )** for a discussion of the relationship between solving the linear system and matrix inversion.

Tolerance

The default tolerance used is

$$\eta = \frac{\texttt{(1e-13)*trace(abs(}G\texttt{))}}{n}$$

where G is the lower-triangular Cholesky factor of A: $n \times n$. A is declared to be singular if `cholesky()` (see [M-5] **cholesky( )**) finds that A is not positive definite or, if A is found to be positive definite, if any diagonal element of G is less than or equal to η. Mathematically, positive definiteness implies that the matrix is not singular. In the numerical method used, two checks are made: `cholesky()` makes one, and then the η rule is applied to ensure numerical stability in the use of the result `cholesky()` returns.

If you specify $tol > 0$, the value you specify is used to multiply η. You may instead specify $tol \leq 0$, and then the negative of the value you specify is used in place of η; see [M-1] **Tolerance**.

See [M-5] **lusolve( )** for a detailed discussion of the issues surrounding solving nearly singular systems. The main point to keep in mind is that if A is ill conditioned, then small changes in A or B can lead to radically large differences in the solution for X.

Conformability

`cholsolve(`*A*`,` *B*`,` *tol*`)`:

input:

A:	$n \times n$	
B:	$n \times k$	
tol:	1×1	(optional)
result:	$n \times k$	

`_cholsolve(`*A*`,` *B*`,` *tol*`)`:

input:

A:	$n \times n$	
B:	$n \times k$	
tol:	1×1	(optional)

output:

A:	0×0
B:	$n \times k$

`cholsolvelapacke(`*A*`,` *B*`,` *tol*`)`:

input:

A:	$n \times n$	
B:	$n \times k$	
tol:	1×1	(optional)
result:	$n \times k$	

`_cholsolvelapacke(`*A*`,` *B*`,` *tol*`)`:

input:

A:	$n \times n$	
B:	$n \times k$	
tol:	1×1	(optional)

output:

A:	0×0
B:	$n \times k$

Diagnostics

`cholsolve(`*A*`,` *B*`, ...)` and `_cholsolve(`*A*`,` *B*`, ...)` return a result of all missing values if *A* is not positive definite or if *A* contains missing values.

`_cholsolve(`*A*`,` *B*`, ...)` also aborts with error if *A* or *B* is a view.

`cholsolvelapacke(`*A*`,` *B*`, ...)` and `_cholsolvelapacke(`*A*`,` *B*`, ...)` return a result of all missing values if *A* is not positive definite or if *A* contains missing values.

`_cholsolvelapacke(`*A*`,` *B*`, ...)` also aborts with error if *A* or *B* is a view.

All functions use the elements from the lower triangle of *A* without checking whether *A* is symmetric or, in the complex case, Hermitian.

Also see

[M-5] **cholesky( )** — Cholesky square-root decomposition

[M-5] **cholinv( )** — Symmetric, positive-definite matrix inversion

[M-5] **lusolve( )** — Solve AX=B for X using LU decomposition

[M-5] **qrsolve( )** — Solve AX=B for X using QR decomposition

[M-5] **solvelower( )** — Solve AX=B for X, A triangular

[M-5] **svsolve( )** — Solve AX=B for X using singular value decomposition

[M-5] **solve_tol( )** — Tolerance used by solvers and inverters

[M-4] **Matrix** — Matrix functions

[M-4] **Solvers** — Functions to solve AX=B and to obtain A inverse

For suggested citations, see the FAQ on citing Stata documentation.