Title

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Intro 4e — Including a second-order lag of a control

Description Remarks and examples Also see

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Some DSGE models capture delayed effects by including a second-order lag of a control variable and excluding the first-order lag. The second-order lag is a problematic term that does not fit into the form required to solve a structural model for its state-space form. This entry shows how to solve this problem by defining new state variables and rewriting the equations.

Remarks and examples

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Remarks are presented under the following headings:

The model

Consider a model in which changes in hours worked take two periods to adjust because next period's hours have already been budgeted. In this model, the second-order lag of changes in hours worked is included, and the first-order lag is excluded. Equations (1)–(4) specify such a model of growth in hours worked and of consumption growth.

$$n_t = b_1 n_{t-2} + w_t - \gamma c_t \tag{1}$$
$$c_t = (1-h) w_t + h E_t c_{t+1} + r_t \tag{2}$$
$$w_{t+1} = \rho w_t + \xi_{t+1} \tag{3}$$
$$r_{t+1} = \epsilon_{t+1} \tag{4}$$

Equation (1) specifies that the growth rate of hours worked n_t depends on a second-order lag of itself, wage growth w_t, and consumption growth c_t. Equation (2) specifies that consumption growth is a linear combination of wage growth, expected future consumption growth $E_t c_{t+1}$, and the interest rate r_t. Equation (3) specifies an autoregressive process for wage growth. Equation (4) specifies that interest rate is just a shock. The control variables are n_t and c_t. The state variables are w_t and r_t.

One cannot solve the model in (1)–(4) for the state-space form because the problematic term $b_1 n_{t-2}$ does not fit into the required form. To accommodate this term, we define two new state variables, one for n_{t-1} and one for n_{t-2}. We define new state variables instead of new control variables because lags of the control are predetermined and thus exogenous. The model with new state variables is

$$n_t = b_1 L2n_t + w_t - \gamma c_t \tag{5}$$
$$c_t = (1-h)w_t + hE_t c_{t+1} + r_t \tag{6}$$
$$w_{t+1} = \rho w_t + \xi_{t+1} \tag{7}$$
$$r_{t+1} = \epsilon_{t+1} \tag{8}$$
$$Ln_{t+1} = n_t \tag{9}$$
$$L2n_{t+1} = Ln_t \tag{10}$$

Equation (9) defines the new state for n_{t-1}, and (10) defines $L2n_t$ to be the new state for n_{t-2}. The $L2n_t$ in (5) replaces n_{t-2} in (1).

Parameter estimation

We specify `n` and `c` as observed control equations. We specify `w`, `r`, `Ln`, and `L2n` as state equations. We specify that `w` and `r` are subject to shocks; the new states to accommodate n_{t-2} are not subject to shocks.

```
. use https://www.stata-press.com/data/r18/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (n = {b1}*L2n + w - {gamma}*c)
>      (c = (1-{h})*w + {h}*F.c + r)
>      (F.w = {rho}*w, state)
>      (F.r = , state)
>      (F.L2n = Ln, state noshock)
>      (F.Ln = n, state noshock)
(setting technique to bfgs)
Iteration 0:  Log likelihood = -2325.1996
Iteration 1:  Log likelihood = -1277.0146  (backed up)
Iteration 2:  Log likelihood = -1193.4512  (backed up)
Iteration 3:  Log likelihood = -1189.3181  (backed up)
Iteration 4:  Log likelihood = -1188.2629  (backed up)
(switching technique to nr)
Iteration 5:  Log likelihood = -1187.9872  (backed up)
Iteration 6:  Log likelihood =  -1147.018
Iteration 7:  Log likelihood = -1131.4022
Iteration 8:  Log likelihood = -1129.0383
Iteration 9:  Log likelihood = -1129.0181
Iteration 10: Log likelihood = -1129.0181
DSGE model
Sample: 1955q1 thru 2015q4                      Number of obs = 244
Log likelihood = -1129.0181
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
b1	.132084	.0608727	2.17	0.030	.0127758	.2513922
gamma	.3609224	.1298382	2.78	0.005	.1064442	.6154007
h	.7238124	.0406724	17.80	0.000	.6440959	.8035289
rho	.6177973	.0533568	11.58	0.000	.5132199	.7223746
sd(e.w)	3.033795	.2423843			2.55873	3.508859
sd(e.r)	1.970291	.1574526			1.66169	2.278893

Looking at the confidence interval for `b1`, we conclude that the second-order lag of hours' growth impacts current hours' growth.

Also see

[DSGE] **Intro 2** — Learning the syntax

[DSGE] **Intro 4** — Writing a DSGE in a solvable form