Logistic Regression Models Joseph M Hilbe

ERRATA and COMMENTS 4th Printing (Printed Sept, 2010) (updated to: 12 December, 2011)

The 4th printing enhances Stata code to use version 11 rather than version 9-10 code. The book was completed before Stata version 11 was published. For example, when constructing synthetic data, the book now uses the new Stata pseudo-random number generators rather than the ones I created back in 1995 – the suite of *rnd** commands -- or Roberto Gutierrez's unpublished *genbinomial* command.

No more corrections to the text are planned for future printings. A second edition is planned to be published in 2013 and will include nested logistic regression, and chapters on latent class models and on Bayesian logistic models. Both single and multilevel models will be examined. Certain areas of the present edition will be re-written to assist in clarity Any suggestions you have, or typos/errors you discover in the present printing of the first edition will be most appreciated.

Instructors may request a gratis 187-page *Solutions Manual for Logistic Regression Models*, Chapman & Hall/CRC, ISBM: 978-1-4398-2066-7. Contact author for details (hilbe@asu.edu or jhilbe@aol.com). It is available from publisher, but I will need to give you added information.

NOTE: 4th Printing is found on page opposite the table of contents. The numbers on the line under "Printed in the United States of America..." end with the number 4 -- the last number is the printing. Thanks to Zhehui Luo of the Michigan State Dept of Epidemiology and students in my courses on Logistic Regression and Advanced Logistic Regression for identifying remaining typos & errors. I have added comments and additions to the actual errata. The Comments section follows the Errata, beginning with page 3.

Page xvii: Final full paragraph at the bottom of the page. The books web site should now read: *http://works.bepress.com/joseph_hilbe/*

Page 1(bottom) page 2 (top): Starting from the sentence beginning with "First, the error term..." on the bottom line of Page 1, amend to read: "First, the error terms are non-normally distributed. Second, the..."

Page 18: The terms A*D/B*C near the bottom of the page: Change to small letters to read as: "The odds ratio is calculated by (a*d)/(b*c) or (a/c)/(b/d)."

Page 19: Near top of page: add "nolog" to first Stata command line under "MAXIMUM LIKELIHOOD LOGISTIC COMMAND". Read as:

". logistic death anterior, nolog"

Page 30,31: The comments to the right of the calculations of probabilities for each of the three non-reference Killip level. Delete the ending phrase for each, "with respect to KK1".

Page 39: The top Stata output in mid page: the term "ons" should read "_cons"

Page 110 Eq 5.19 Close parentheses for both numerator an denominator.

Page 118: ">" sign between RR and left side equation should be "="

Pages 120 and 128: The "///" symbols should be "//".

Page 130 third word, "percent", of the first full paragraph is misspelled. The sentence should read as: "The 95 percent confidence interval of the attributable risk is given as"

Page 132 third/fourth line under equation 5.40. Change sentence beginning with "Scaling replaces" to read as:

"... Scaling replaces *W* by the product of the model standard error and square root of the Pearson dispersion statistic."

Thus, scaled SE = $se(\beta s) = se(\beta)*sqrt(Pdispersion)$.

Page 133: Close space between *rbinomial* and (*d*,*exb*). Read as gen by = rbinomial(d, exb)

Page 172: R code: 9th line from top of page. Should read as:

age2 <- ifelse(agegrp=='61-69', 1,0)</pre>

page 191: Stata code near bottom of page: Replace "of" with "if" to read:

. drop if class==4

Page 215: Amend equations 6.11 and 6.12 so that there is a bracket on the 3rd term of each

```
Variance = (r_1 - r_0)^2 * V(\beta_1) + [x(r_1 - r_0)]^2 * V(\beta_3) + 2x(r_1 - r_0)^2 * CV(\beta_1, \beta_3) (6.11)
SE = sqrt[(r_1 - r_0)^2 * V(\beta_1) + [x(r_1 - r_0)]^2 * V(\beta_3) + 2x(r_1 - r_0)^2 * CV(\beta_1, \beta_3)] (6.12)
```

Page 217: The formula used to calculate a p-value near the bottom of the page is mistaken. See page 104 for explanation. The last Stata code and output on the page should read as:

. di (1-normprob(1.404184))*2 .16026407

The corresponding R code is (for pages 239/240)

> pnorm(1.40184, lower.tail=F)*2

Page 219, Figure 6.4 Stata's graph commands have changed since first written. New code is:

Page 220, Figure 6.5 Stata's graph commands have changed since first written. New code is:

```
. scatter yhat0 yhat1 los, connect(l l) symbol(0 d) xlabel(0 10 to 100)
    sort l1title(Predicted logit) title(Interaction of White and LOS)
```

Page 227: 5 lines from the bottom, first term and number in line. "80" should read "90".

Page 259. Add sentence to the end of Section 7.3, just above 7.3.1 "In general, BIC statistics give greater adjustment weight to the number of predictors in the model than does AIC. "

Page 263: Section 7.3.3-7.3.5 to be amended to read as follows. Substitute the text between the double-double lines for what is now in the book. My apologies for the inconvenience. Delete the current section 7.3.3 LIMDEP AIC. It is appropriate for normal models, not logistic models.

7.3.3 Other AIC statistics

There have been a number of AIC-type statistics developed since Akaike first constructed his information criterion in 1973. Two others that have found considerable use are both called corrected AIC statistics. The first was by Sigiura (1978), formulated as

$$AIC_c = AIC + \frac{n(p+1)}{n-p-2}$$

(7.24)

Simulation studies have shown it to have less bias than the AIC, and to perform better than AIC when n/p is small.

The second corrected version was by Bozdogan (1987). He defined the equation as

$$CAIC = -2L + p\{ln(n) + 1\}$$

(7.25)

Bozdogan criticized Akaike's original formulation of AIC due to the fact that it does not depend on sample size. Because of this he showed that it lacked the properties of asymptotic consistency. Sigiura's definition addresses the same problem. Studies have demonstrated that AIC_c in particular is preferred to AIC for assessing comparative model fit. We use AIC_n instead for most of our comparative analyses. It appears to be able to select the best fitted model as well as the statistic with more terms..

7.3.4 BAYESIAN INFORMATION CRITERION (BIC)

The BIC statistic was first developed by Gideon E. Schwarz of Hebrew University, Jerusalem, in 1978. His formulation was in response to Akaike's 1973 information criterion, whereby more weight is given to the number of predictors in the model. Schwarz also included a term for

sample size, which all subsequent formulations of AIC or BIC after the original AIC have done. The philosophical basis of Schwarz's information criterion is Bayesian, unlike Akaike, but the resulting AIC and BIC equations have typically differed by only a term or so. The rationale for the two types of information criteria differ, but the resulting formulae are similar. The reasons for this go beyond the scope of this book.

Schwarz's Bayesian Information Criterion (BIC), also referred to as simply *Schwarz Criterion* in SAS output, is given as

$$BIC_{S} = -2LL + k^{*}ln(n)$$
 (7.26)

The model with a lower BIC statistic is regarded as the better fitted model. The models being compared may be nested, but need not be. The models may be of different sample sizes as well. Comparisons of treatment and control data are common applications of the BIC statistic.

For an example, we model the same predictors as before, but use the GLM logistic model. I follow estimation with a command called *abic*, which produces two AIC statistics and two BIC statistics. The AIC-BIC pair in the right column are the statistics that are commonly displayed following Stata maximum likelihood estimation. The Stata command, *estat ic*, displays these statistics to the screen.

```
. glm death anterior hcabg kk2-kk4 age3 age4, nolog fam(bin) eform
```

Generalized li Optimization		No. c Resid	No. of obs Residual df		4503 4495 1		
Deviance Pearson	= 1276.31 = 4212.63	= 1276.319134 = 4212.631591		(1/df (1/df	1/df) Deviance 1/df) Pearson	=	.283942 .9371817
Variance funct Link function	: g(u) = u : g(u) = 1	*(1-u) n(u/(1-u))		[Bern [Logi	oulli] t]		
Log likelihood	a = -638.159	5669		AIC BIC		=	.2869907 -36537.86
 death	Odds Ratio	OIM Std. Err.		P> z	[95% Conf		Interval]
anterior hcabg kk2 kk3 kk4 age3 age4	1.894096 2.195519 2.281692 2.218199 14.63984 3.549577 6.964847	.3173418 .7744623 .4117012 .5971764 5.218374 .7119235 1.44901	3.81 2.23 4.57 2.96 7.53 6.32 9.33	0.000 0.026 0.000 0.003 0.000 0.000 0.000 0.000	1.363922 1.099711 1.602024 1.308708 7.279897 2.395823 4.632573		2.630356 4.383246 3.249714 3.759743 29.44064 5.258942 10.47131
. abic AIC Statistic BIC Statistic	= .2869907 = .2908262		AIC*n BIC(Stat	= 1292	.3191		

Notice that the model output includes an AIC and BIC statistic. The displayed BIC statistic is - 36537.86, which is different from Stata's BIC which is displayed in the *abic* output --- with a value of 1343.6191. The value of AIC in the model output is the same as indicated by AIC*n.

The model BIC is based on the Deviance definition, and is expressed as

$$BIC_{R} = D - df * ln(n)$$
(7.27)

We may calculate it as follows,

```
di 1276.319134 - 4495*ln(4503) // LL - dof*ln(n) -36537.864
```

which gives the same value as shown in the above model output. This version of BIC was specifically designed by the University of Washington's Adrian Raftery in 1986 to be used with GLM software. GLM algorithms generally base model convergence on the deviance function, and a few applications do not even estimate a log-likelihood function during the modeling process. The BIC_R statistic is given rather than Schwarz's statistics due to the GLM estimation environment. *abic* provides Schwarz's criterion, giving us both versions to be used for comparative analysis.

The BIC_S statistic may be calculated as

. di -2*(-638.15957) + 8 * ln(4503) 1343.6191

which is identical to the value displayed in *abic* results. The *abic* right column statistics can be produced using Stata's *estat ic* command.

•	estat ic						
	Model	Obs	ll(null)	ll(model)	df	AIC	BIC
	.	4503		-638.1596	8	1292.319	1343.619
		Note:	N=Obs used in	calculating	BIC; see	[R] BIC note	

Raftery developed a table providing the degree of model preference based on the absolute difference between the BIC statistics of two models.

|difference|Degree of preference0 - 2Weak2 - 8Positive6 - 10Strong> 10Very Strong

Models A and B:

If $BIC_A - BIC_B < 0$, then A preferred If $BIC_A - BIC_B > 0$, then B preferred or

Model with lower BIC value preferred.

For the example models we have worked with in this chapter, the reduced model has the following partial output.

	No. of obs	=	4503
	Residual df	=	4497
	AIC	=	.3074784
Log likelihood = -686.2875063	BIC	= -	-36458.43
. abic			
AIC Statistic = .3074783	AIC*n = 1384.5751		
BIC Statistic = .3095883	BIC(Stata) = 1423.05		
The deviance based BIC statistics are	: -36537.86 to -36458.43		
The log-likelihood based BIC statistics are:	1343.62 to 1423.05		
DEVIANCE			
. di -36537.86 -(-36458.43) -79.43			

```
LOG-LIKELIHOOD
. di 1343.62 - 1423.05
-79.43
```

Both differences are identical. This relationships maintains for other nested models as well. In either case, however, the absolute difference between the full and reduced model is substantially greater than 10, indicating a very strong preference for the full model.

The AIC and BIC statistics give us consistent advise. Both the AIC and BIC tests tell us that the full model is preferred.

Recall that the true value of the AIC and BIC statistics rests in the fact that they can both compare non-nested models. For example, modeling the same data using the full model with a Bernoulli *loglog* link, the likelihood BIC statistic is 1335.4843. Compare this to the same statistic *logit* link value of 1343.6191. The difference is 8.13, in favor of the *loglog* link. This indicates that the *loglog* link is strongly preferred over the *logit* link.

Note that the deviance statistic is used only for GLM-based statistical procedures. The loglikelihood is the normal way to estimate all other maximum likelihood models. Most contemporary GLM algorithms have, though, a calculated log-likelihood function as part of the output, therefore the deviance-based formula is rarely used now.

7.3.5 HQIC GOODNESS OF FIT STATISTIC

The HQIC, or Hannan and Quinn Information Criterion (Hannan & Quinn, 1979), is defined as

$$HQIC = -2\{LL - k^* \ln(k)\}/n$$
(7.28)

The calculated values for the nested models we have been discussing in this chapter are:

```
. di -2*(-638.15957 -8*ln(8))/4503
.29082616
. di -2*(-686.28751 -6*ln(6))/4503
.3095883
```

The HQIC test is an alternate version of BIC, used in LIMDEP. If the values of the BIC and HQIC differ greatly, it is wise to check the models.

It is vital to be certain you know which version of AIC and BIC is being used. Whatever the version, be consistent throughout the comparative evaluation of models.

7.3.6 A Unified AIC Fit Statistic

Page 272. Equation 7.31, Parentheses are needed for the denominator of the second term within brackets, $y/(m^*\mu)$). The equation should read as:

 $d = +/- \operatorname{sqrt}[2\Sigma y * \ln(y/(m*\mu)) - (m-y)*\ln((m-y)/(m*(1-\mu)))]$

Page 293. R code: 3rd line from top.Use:library(PresenceAbsence)in place of:library(epicalc)

Page 299; Equation 8.14: the final term should read $ln\binom{m}{v}$.

Page 300: Code in mid-page. Should read as: $\chi^2 = \Sigma (y-\mu)^2 / (\mu^* (1-\mu/m))$ LL = $\Sigma \{ y^* \ln (\mu/m) + (m-y)^* \ln (1-(\mu/m)) \}$

Page 323: Delete "/// a user authorized command" near the top right of the page.

Page 335: Delete the "]" at the end of the long line of Stata code in middle of page.

Page 368: Box 10.1: the section on *white* should read as *white*: The expected odds of being admitted to the hospital as an emergency patient is some 40 % less among those who identified themselves as white compared with those who identified themselves as non-white, holding the other predictors constant

Page 376: Line immediately above Section 10.4: change words "a higher level" to "Emergency".

Page 387: The first word, "The", of the paragraph immediately under equation 11.9 is mistaken. The paragraph should start out as:

"It is important to remember that the above parameterization is based on set-"

Page 388: the table about ¹/₄ a page from the top has the 0 and 1 values in the wrong places. It should instead read as:

		Response			
			0	1	
Predictor	0		А	B	
	1		С	D	

If you find additional errata, please advise. I will post them to this Errata page in the future. Thank you to those who have identified typos. I will list your names in the second edition.

REFERENCES

p 621: Replace the current reference for Swartz, J to read as: Schwarz, G (1978). Estimating the dimension of a model, *Annals of Statistics*, Vol 6, 2:461-464.

Add:

Bozdogan, H. (1987). Model-selection and Akaike's information criterion (AIC): The general theory and its analytical extensions. *Psychometrika*, 52, 345-370.

Sugiura, N (1978). Further analysis of the data by Akaike's information criterion and the finite corrections. *Communications in Statistics*. A 7, 13-26.

COMMENTS

Page 65: I probably should have added the formula for the second derivative of the Bernoulli link function under Equation 4.12.

$$g''(\mu) = (2\mu - 1) \left(\frac{\partial \eta}{\partial \mu}\right)^2 = \frac{(2\mu - 1)}{\mu^2 (1 - \mu)^2}$$

(4.12a)

Page 67: Table 4.1 provides a schematic algorithm for the estimation of a binary logistic model. I have provided full working code for estimating a generic logistic regression using Stata and R. I display the code and output below for each below the final Comment in this section.

Page 132 Comment: In R, the *glm quasibinomial* family is the same as scaling the binomial logistic model standard errors by the Pearson dispersion statistic. I recently discovered that the R *vcov()* function that is used by programmers for calculating standard errors in fact creates scaled standard errors. This results in the SEs of models using *sqrt(diag(vcov(modelname)))* for calculating SEs to have different SEs from Stata, SAS, and other applications, particularly when the data is correlated. Dividing the displayed SEs by the square root of the Pearson dispersion statistic produces model SEs. R's *glm()* function, which is used for estimating both binary and grouped logistic models, adjusts SEs so that model SEs are displayed in the results.

Page 300 Suggestion: the deviance function as presented is the standard one shown in texts. However, it does not work properly if used in an R GLM program. A much more simple and suitable expression for the equation, requiring less memory, is the following:

Dev = $2\Sigma \{y^{1} (1/\mu) + (m-y)^{1} (1/(m-\mu)) \}$

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STATA USER AUTHORED LOGIT COMMAND. First published in the November 2005 issue of *The American Statistician* in a review of Stata. The review may be obtained from my BePress Selected Works site, http://works.bepress.com/joseph_hilbe/

```
*! version 1: LOGISTIC REGRESSION :IRLS METHOD OF ESTIMATION
* Joseph Hilbe: TAS - Stata 9.0 review: 7Jul2005
program define jhlogit
version 6
set type double
syntax varlist(default=none) [if] [, EForm]
gettoken y varlist : varlist
if `"`if"'' != `""' {
    preserve
                      /* ensure the dataset returns at end of pgm */
    keep `if'
                     /* retain only estimation sample */
ł
if "`eform"' != "" { local eform "eform(Odds Ratio)" }
qui {
tempvar mu eta u w z dev oldev llike chi2 aic bic
* INITIALIZATION OF MU AND ETA
count
local nobs = _result(1)
gen mu' = (y' + 0.5)/2
gen eta' = \ln(mu'/(1-mu'))
* VARIABLE INITIALIZATION
local i
        1
gen `u' =0
gen `w' =0
gen z' = 0
gen `dev' =1
gen `oldev'=1
gen `chi2' =1
local ddev 1
* IRLS SCORING
while (abs(`ddev')> 1e-6 ) {
replace u' = (y'-mu')/(mu'*(1-mu'))
replace w' = mu'*(1-mu')
replace z' = eta' + u'
regress `z' `varlist' [iw=`w'], mse1 dep(`y')
drop `eta'
predict `eta'
replace mu' = 1/(1+exp(-eta'))
replace `oldev'= `dev'
replace dev' = \ln(1/mu') if y'==1
replace dev' = \ln(1/(1-mu')) if y'==0
```

```
replace `dev' = sum(`dev')
replace dev' = 2* dev'[N]
local ddev = `dev' - `oldev'
local i = i'+1
}
local npred = _result(3)
                         /* number of predictors */
local df = `nobs' - `npred' - 1 /* degrees of freedom */
* CALCULATION OF LOG-LIKELIHOOD AND GOF STATISTICS
egen `llike' = sum(`y'*ln(`mu')+(1-`y')*ln(1-`mu'))
gen `aic' = (-2*`llike' + 2*`npred')/ nobs' // AIC/observations
}
* PUT VALUES INTO MATRIX
qui regress, noheader `eform'
tempname b V
mat `b' = get(_b) /* coefficient vector */
mat `V' = get(VCE) /* variance-covariance matrix */
mat post `b' `V', depname(`y') obs(`nobs')
* OUTPUT
di " "
di in gr "Logistic Estimates"
mat mlout, 'eform'
di in gr col(1) "Observations = " in ye `nobs' in gr col(53) "Deviance = " in ye `dev'
di in gr _col(53) "Loglikelihood = " in ye `llike'
di in gr _col(1) "AIC Statistic = " in ye `aic'
set type double
end
      _____
```

USE OF COMMAND

R -- Bernoulli or binary logistic regression -

```
# BINARY LOGISTIC REGRESSION. BASIC FUNCTION 7 July, 2011
# From: Hilbe, J.M and A.P Robinson (2012), Methods of Statistical Model
# Estimation, Chapman & Hall/CRC
irls_logit <- function(formula, data, tol=.000001) { # irls_logit options
mf <- model.frame(formula, data)  # define model frame as mf
y <- model.response(mf, "numeric")  # set model response as y
X <- model.matrix(formula, data = data)  # predictors in matrix X</pre>
```

```
if (any(is.na(cbind(y, X)))) stop("Some data are missing.")
mu < - (y + .5)/2
                                             # initialize µ
eta <-\log(mu/(1-mu))
                                              # initialize η
dev <- 2 * sum( y*log(1/mu) + (1 - y)* log(1/(1-mu)) )
deltad <-1
                                              # initialize deltad = 1
i <- 1
                                                # initialize i=1
while (abs(deltad) > tol ) {
                                                # IRLS loop begin
 w <- mu*(1-mu)
                                               # weight
  z <- eta + (y - mu)/w
                                               # working response
 mod < - lm(z ~ X-1, weights=w)
                                               # weighted regression
  eta <- mod$fit
                                                # linear predictor
 mu < -1/(1+exp(-eta))
                                                # fitted value; probability
  dev.old <- dev
                                                # setup for convergence
  dev <- 2 * sum( y*log(1/mu) + (1 - y)* log(1/(1-mu))) # deviance
  deltad <- dev - dev.old
                                                # test of 2 iterations
  cat(i, coef(mod), deltad, "\n")
                                                # iteration log
  i <- i + 1
                                                # recalibrate iter number
}
  beta <- mod$coef</pre>
                                                    # save coefficients
  pr <- sum(residuals(mod, type="pearson")^2) # calc Pearson disp</pre>
  prdisp <- pr/mod$df.residual</pre>
  return(list(coef = coef(mod),
                                                   # coef & SE display
           se = sqrt(diag(vcov(mod))) / sqrt(prdisp)))
```

USE -- how *source()* is defined is based on where *irls_logit.r* is stored on your computer. It will be a function in the *msme* library later in 2011 (download fom CRAN).

}

Coefficients and model standard errors are displayed. Confidence Intervals, Z statistic, and p-values can be easily calculated. Note that the scaled SEs calculated by vcov() are amended to produce true model SEs.

NOTE: A complete description of OLS, IRLS, maximum likelihood, EM, quadrature, simulation, and other major methods of estimation can be found in **Hilbe, Joseph M. and Andrew P. Robinson (2012)**, *Methods of Statistical Model Estimation*, **Chapman & Hall/CRC**. The *irls_logit* function is fully described as an example of IRLS estimation. Other more complex IRLS models are also discussed. In addition, we created a *glm*-like function called *irls*, which corrects what we believe to be shortcomings in *glm()* and *glm.nb()*, describing its modular logic the specifics of the code. After the *msme* library is loaded, *irls()* will be able to be used like *glm()* is now, together with a *summary()* function. *irls()*, however, provides a much more extensive list of post-estimation statistics.

COEFFICIENTS and STANDARD ERRORS > i.logit X(Intercept) Xhmo Xwhite -0.92618620 -0.01224648 0.30338724 Śse X(Intercept) Xhmo 0.1973903 0.1489251 Xwhite 0.2051795 LOWER 95% CONFIDENCE INTERVAL > i.logit\$coef - 1.96*i.logit\$se X(Intercept) Xhmo Xwhite -1.31346050 -0.30443326 -0.09916926 UPPER 95% CONFIDENCE INTERVAL > i.logit\$coef + 1.96*i.logit\$se X(Intercept) Xhmo Xwhite -0.5389119 0.2799403 0.7059437

The Z-statistic and P-values may be easily calculated from the above, but the confidence intervals will indicate if a predictor is significant as well. When odds ratios are displayed, $exp(i.logit\coef)$, recall that the standard errors are determined using the delta method, which in this case is quite simple: $exp(\beta)$ *se(β); ie.

ORse <- exp(i.logit\$coef)* i.logit\$se.

See page 35 in text.

Also of possible interest to readers, James Hardin and I have finished writing the third edition of *Generalized Linear Models and Extensions* (Stata Press) [GLME3]. We are now working on the second edition of *Generalized Estimating Equations* (Chapman & Hall/CRC) [GEE2]. I expect GLME3 to be published about December 15 (2011) We do not have a due date for GEE2, but intend to complete it before Christmas of this year. I am also working on a book with Justine Shults titled *Quasi-Least Squares Regression: Extended GEE methodology* (Chapman & Hall/CRC). It was begun in 2007 and should be completed in early 2012. I discuss QLS and mention the book in LRM (pages 470-480). Also, as mentioned above Andrew Robinson and I are working on *Methods of Statistical Model Estimation* (Chapman & Hall/CRC). It should be finished about March 1, 2012. I am contracted to write other books as well, but will not begin on them until the above are completed.

Thanks to Dongjae Jung for identifying several typos 4 November, 2011.